Growth and Welfare Implications of Sector-Specific Innovations*

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January 11, 2021

Abstract

I examine the optimal government subsidy of R&D activities when sectors are heterogeneous. To this end, I build an endogenous growth model where R&D drives macroeconomic growth and firm dynamics in two sectors with different characteristics: a consumption-goods sector and an investment-goods sector. I highlight how various externalities in the innovation process affect the allocation of innovative resources across industries. I calibrate the model to U.S. data and study the quantitative properties of the model. By explicitly examining the transition path after the change in subsidy, I highlight the tradeoff between the short-run level of consumption and long-run growth. I find that the optimal combination of the subsidy rates, as a fraction of firm R&D expenditures, is 84 percent in consumption sector and 88 percent in investment sector. By moving from the baseline subsidy rates (10 percent in both sectors), society can achieve a 20 percent welfare gain in consumption equivalent terms. The annual GDP growth rate increases from 1.5 percent to 3.2 percent by this change in subsidy. Finally, I show that it is always optimal to subsidize R&D spending in investment sector at a higher rate than that in the consumption sector when the government's subsidy budget is limited.

Keywords: Endogenous Growth, Innovation, Research and Development, Investment Specific Technological Change

JEL classification: O31, O38, O41, L16

*I would like to thank my dissertation committee: Toshihiko Mukoyama, Eric Young, Sophie Osotimehin and Latchezar Popov for their invaluable guidance in this research. I also would like to thank Abiy Teshome, Nate Pattison, Allison Oldham Luedtke, Selcen Çakır, Miguel León-Ledesma, Zach Bethune, Nick Embrey and two anonymous referees for their helpful comments in various stages of this research. I gratefully acknowledge funding from Bankard Pre-doctoral Fellowship fund of the University of Virginia.

1 Introduction

Government support for business research and development (R&D) varies across the globe. In 2011, the United States federal government's total support to business R&D was 0.26% of its GDP, compared to 0.43% in Korea and 0.01% in Mexico [OECD (2015)]. Inter-country variation in government support for R&D suggests that setting the optimal amount of support is complex, potentially varying across sectors. In this paper, I quantify the optimal amount of government subsidy to business R&D.

I build an endogenous growth model with firm dynamics to analyze the optimal R&D subsidy. I use the model and firm dynamics data to identify inefficiencies in the R&D expenditures of two sectors that have different characteristics: the consumption-goods sector and the investment-goods sector. Next, I characterize the subsidy rates required to address inefficiencies in innovation in these sectors. I base my model on the seminal work of Klette and Kortum (2004), where innovation by incumbent and entering firms generates firm dynamics and drives macroeconomic growth. Klette and Kortum show that their model can qualitatively account for various stylized facts about firm dynamics. Hence, I identify the inefficiencies in firm R&D expenditures within a framework that has a good fit on the firm dynamics data. I extend the Klette and Kortum model by introducing capital stock and by having not only a consumption-goods sector, but also an investment-goods sector. Differences in the rates of firm entry and expansion across these sectors and the sustained decline in the prices of investment goods relative to consumption goods imply that in these sectors there are different magnitudes of inefficiency and rates of technological change.[1] A model that recognizes the heterogeneity of innovative activity across sectors will be more informative about the growth and welfare implications of R&D subsidies.

In this paper, an innovation is modeled as an increase in the quality of an existing good in the market along a quality ladder. The consumption-goods and investment-goods sectors consist of many differentiated products, each produced by a production line. In this setting, a firm is simply a collection of the production lines it possess. An innovating firm captures the market of the innovated product from the existing producer and earns monopoly rents for as long as the innovator hold the blueprints of the highest quality versions of the goods it produces. The firm loses these rents following innovation on the same goods by other firms. Firms expand and shrink according to this creative-destruction process, entering the market when they successfully innovate and exiting if they lose the blueprints of all the goods they produce.

The model summarizes R&D activity in the economy through the *innovation rate* in each sector, defined as the ratio of the measure of differentiated products innovated at an instant to the measure of total products in that sector. The creative-destruction process associated with the market economy generates innovation rates that differ from the socially optimal innovation rates in each sector. Such inefficiencies arise due

[1]See Gordon (1990) and Cummins and Violante (2002) for the decline the in the relative price of investment goods.

to differences between the innovator's objective and societal welfare. The innovator reaps monopoly rents as long as it holds the blueprints of the highest-quality version of the product she innovated. In contrast, society benefits from innovation due to the extra production from the innovation. Relatedly, in contrast to the limited lifetime of monopoly rents accruing to the innovator, the social benefit of any given innovation lasts forever. There are four differences between the innovator's objective and social welfare. On the one hand, an entrepreneur's inability to appropriate all of the consumer surplus it creates causes market economy innovation rates to fall below those of social planner levels (*the appropriability effect*). Also, the limited time of monopoly rents accruing to the innovator contributes to under investment in innovation (*the inter-temporal spillover effect*). On the other hand, an entrepreneur does not account for the profit loss imposed on the current producer of the product that it takes over, and this moves market innovation rates above the socially efficient level (*the business-stealing effect).* Finally, the monopoly power of the innovator causes factor prices to differ from the marginal products of the factors of production, and leads to over investment in innovation in the market economy (*the monopoly-distortion effect*).[2] Depending on the sizes of these distortions, market economy innovation rates can be below or above the socially efficient levels. Therefore, a government may wish to employ tax/subsidy systems to correct the distortions in the economy, thereby increasing household welfare.

These distortions lead not only to deviations from the socially optimal levels of industry innovation rates but also to the misallocation of innovative resources across sectors. In particular, holding the total R&D labor constant at the competitive equilibrium level, the social planner can generate welfare gains by reallocating research labor across industries. We observe such misallocations even when the two sectors have identical innovation functions, suggesting that sectors vary in their exposure to these distortions based purely on their location in the supply chain. Put differently, the externalities distort the allocation of research labor in (potentially) different directions. For example, the monopoly power effect tilts innovation rates toward the consumption sector, while the inter-temporal spillover effect pushes innovation rates toward the industry with the lower socially optimal innovation rate. The former is a result of relative profits in the consumption sector exceeding the relative effectiveness—in the sense of promoting economic growth—of innovation in the consumption sector. The latter is a result of the inter-temporal spillover effect being larger in the industry with the higher innovation rate. In an environment with identical innovation functions across industries, the inter-temporal spillover effect distorts allocation of research labor toward the investment sector.

The optimal amount of subsidy for each sector depends on the elasticity of R&D with respect to the subsidy and the magnitudes of externalities in each sector. Various studies have estimated the former using data on firm level R&D expenditures and changes in government subsidy rates [Bloom et al. (2002), Hall et al. (2010), CBO

[2] I use the terminology of Aghion and Howitt (1992) to describe the distortions in the innovation process.

(2005)]. In contrast, the magnitudes of externalities are not observable. To infer the sizes of the externalities in each sector and devise the optimal R&D subsidy system, I identify model parameters related to innovation that are obtained from the model's implications on firm dynamics. Recent literature emphasizes that the firm dynamics data contains important information on the innovation process. For example, Klette and Kortum (2004) qualitatively generates many empirical facts on the firm size distribution and firm growth rates. Lentz and Mortensen (2008), whose model is based on Klette and Kortum, estimate model parameters from Danish data, and the model quantitatively fits related firm dynamics moments.

Differences in the firm dynamics of the two sectors, as observed in the US data, are consistent with different magnitudes of externalities in the two sectors. First, the model relates the expected lifetime of monopoly rents that accrue to an innovator after successful innovation to the entry rate in the firm's sector (the inter-temporal spillover). Hence, a high observed entry rate in the consumption sector suggests a large inter-temporal spillover effect. Second, the business-stealing effect is driven by the difference between the profit an innovating firm captures and the *net benefit of innovation to society*, the extra production generated by the innovation, net of the profits lost by the incumbent producer. The model relates this additional innovation-induced production to the size of the quality improvement (the quality ladder step size), and relates the quality ladder step size to the GDP growth rate and the growth rate of the investment goods price relative to the consumption goods price. Hence, a high observed consumption growth rate suggests a large quality ladder step size in both industries; a high observed growth rate in the relative price of the investment goods (in absolute value) suggests a larger quality ladder step size in the investment sector than in the consumption sector; and a large quality ladder step size suggests a small business-stealing effect. The calibration exercise shows that the consumption-goods sector has lower quality ladder step size than the investment-goods sector, suggesting a larger business-stealing effect in the consumption-goods sector. In related work, Ngai and Samaniego (2011) study US data in a multi-sector endogenous growth setting, and document heterogeneity in innovation and production functions, and in consumer preferences across different investment good producing industries. In contrast, I focus on the heterogeneity between the consumption goods and investment goods sectors, and conduct optimal R&D subsidy analysis.

As explained above, market innovation rates are inefficient. I gauge the degree of under- or over-investment in innovation by solving the social planner problem, where a social planner dictates firms' R&D and production decisions subject to their innovation functions. Over the long-run, the social planner sets innovation rates that are substantially higher than the market rates in each sector: 13 percentage points higher in the consumption industry and 17 percentage points higher in the investment industry. Over the long term, the increased innovation rates correspond to a 1.7 percentage point increase in the GDP growth rate. Starting from the balanced growth path of the market economy with 1.5% GDP growth rate, the social planner immediately decreases the GDP growth rate to less than 1 percent, before the growth rate gradually increases to its new balanced growth path value of 3.2 percent. However, consumption and GDP follow different trajectories. Like the consumption growth rate, the level of consumption decreases initially as more labor is employed in research. Although the consumption growth rate increases gradually, it remains below its market economy balanced growth path level for some time. Eventually, the consumption growth rate converges with its balanced growth path level of 3.2 percent. Long-run consumption growth outweighs the short-run consumption loss, and the transition from the market economy balanced growth path to the social planner balanced path leads to an almost 21.5 percent welfare gain to households, as measured in consumption–equivalent terms. Thus, the market is under-investing in innovation, and a benevolent government can increase the household welfare by subsidizing R&D.

The amount of resources allocated to R&D at the social planner's balanced growth path is about six times the market economy resource allocation to innovation. This is larger than the levels reported in recent articles that employ related models but different methods. For example, Lentz and Mortensen (2008), estimated on Danish data, consider a setting where firms differ persistently in their ability to create higher quality products. Based on the estimates from this paper, Lentz and Mortensen (2015) show that the social planner would increase resource allocation to innovation threefold compared to market outcome, thereby generating a 21 percent welfare gain, as measured in the tax to social planner consumption. My estimated 21 percent welfare gain accounts for the transition path, a factor omitted in the Lentz and Mortensen analysis, which calculates the welfare gain by comparing the steady states of the market and the social planner economy.

I consider government intervention in a decentralized environment through subsidies to R&D activity and entry, all financed by lump-sum taxation of households. In my benchmark calibration, subsidizing 84 percent of consumption sector incumbents' R&D expenditures and 88 percent of investment sector incumbents' R&D expenditures generates a welfare gain (20.3 percent) close to that achieved by the social planner. In addition, the government employs an entry subsidy to equate the marginal social cost of entrant innovation and the marginal social cost of incumbent innovation. This result suggests that government can substantially increase the societal welfare by heavily subsidizing innovation.

Although replicating the socially optimal outcome may, in principle, require time-varying subsidy rates, my analysis reveals that a time-invariant subsidy system generates welfare gains that are comparable to those from moving to the social planner's allocation. Similarly, Grossmann et al. (2013) calculate socially optimum time-dependent R&D subsidy rate and report negligible welfare losses from setting instantaneously the R&D subsidy rate at its long-run value rather than employing a time-varying R&D subsidy rate. They also show that the optimal R&D subsidy is approximately 81.5%. Both results are in line with my findings. Akcigit et al. (2016) address optimal R&D policy in an environment with firm heterogeneity in research productivity and asymmetric information about this research productivity. They show that the optimal subsidy system depends on many factors, including the age of innovating firms, and the current and lagged product quality and R&D expenditures of the firms. Since there are no information asymmetries in my model and per-good research productivity is constant across firms within a sector, the optimal R&D subsidy according to Akcigit et al. (2016) would be constant across all firms in any given sector.

Subsidizing incumbent firms' R&D expenditures to maximize welfare gains requires taxes on the order of 16 percent of GDP. For reasons exogenous to my model, such large transfers to businesses might not be attainable. With this in mind, I calculate the optimal government R&D subsidy to industries when the government's transfer budget is constrained by some exogenous factors. If the government can increase its R&D subsidy budget as a share of GDP by 10 percent, it can achieve a 0.2 percent welfare gain by taxing the consumption sector incumbents' R&D expenditures by 1.1 percent and subsidizing the investment sector incumbents' R&D by 15.3 percent. I repeat this exercise with different transfer budget constraints and find that it is always optimal to subsidize the investment sector at a higher rate than the consumption sector. In a similar exercise, I show that if a social planner is allowed to increase R&D labor by 10 percent, she can achieve a welfare gain of 2 percent by allocating 15.6 percent innovation rate to the investment sector and 14.5 percent innovation rate to the consumption sector. Therefore, there is a non-negligible welfare gain of subsidizing R&D even if there are limits to resources allocatable to innovation.

Comparing my model with the Atkeson and Burstein (2019) (AB hereafter) model highlights the main contribution of this research, the analysis of optimal innovation subsidies in an environment with heterogenous sectors. AB build a model that nests many endogenous and semi-endogenous growth models. They develop a method to linearly approximate output and productivity trajectories after a policy-induced change in the innovation intensity of the economy. Their methodology relies on a few critical assumptions, one of which is conditional efficiency: any given level of innovative resources in the economy is efficiently allocated across sectors. Whenever this assumption does not hold, their analysis applies only to proportional changes in innovation subsidies to different agents in the economy. As explained above, this key assumption does not hold in my model. Further, my methodology applies to non-proportional changes in subsidies to firms in different industries. To be clear, under the assumptions of the AB model, the AB method approximates output and productivity trajectories quite well with more politically feasible innovation policies (a 10% increase in policy-induced changes in R&D labor). AB also have a richer model in other aspects; to name a few, different levels of inter-temporal spillovers, technological progress with quality ladders as well as technological progress with expanding varieties, and so on.

Subsidizing only the investment sector produces a larger welfare gain than subsidizing only the consumption sector. Two opposing factors contribute to the difference in welfare gains. First, although each sector has a similar elasticity of innovation with respect to the user cost of R&D, the investment sector has a higher innovative step. Hence, any decrease in the user cost would lead to similar changes in innovation rates, but a given change in investment sector innovation leads to a higher consumption growth rate and, hence, a larger welfare gain than a similar change in consumption sector innovation. Second, an increase in the investment sector innovation rate leads to a larger reduction in the price of investment goods, which, in turn, increases the user cost of capital, resulting in a lower accumulation of capital. Consequently, consumption production grows more slowly than it would otherwise. During earlier periods, the investment-sector-subsidized economy has a lower consumption than the consumption-sector-subsidized economy. Overall, the first factor dominates and the welfare gain of subsidizing investment sector R&D is higher.

To achieve welfare-maximizing innovation rates, the government needs to subsidize innovation at roughly 85 percent. This large subsidy rate follows largely from two factors. First, there are significant distortions in the economy. As explained above, using related models, Atkeson and Burstein (2019) and Lentz and Mortensen (2015) find substantial under-investment in innovation in the market economy. Similarly, Jones and Williams (2000) find that the market economy typically under-invests in innovation. Second, R&D subsidies encourage innovation by decreasing the cost of innovation, but they also discourage incumbent innovation by reducing the expected lifetime of an innovation. A higher subsidy leads to a higher firm value, which increases the entry rate. When the entry rate increases, an incumbent firm is more likely to lose its monopoly rents by successfully innovating, and this reduces the expected time period of monopoly rents and the value of innovation (inter-temporal substitution effect increases). Thus, innovation will be discouraged. To compensate for the shortened expected lifetime of innovation, firms need to be subsidized even more.

This paper is organized as follows. Section (2) describes the model while Section (3) theoretically characterizes distortions in the economy and analyzes the impacts of the distortions on the allocation of innovative resources across sectors. Section (4) calibrates the model. Section (5) numerically compares market outcome to the social planner's equilibrium. Section (6) characterizes the subsidy system that would maximize household welfare. Section (7) compares my results with the results of Atkeson and Burstein (2019) and highlights my contributions. Section (8) concludes.

2 Model

Time is continuous. There are two sectors in the economy: consumption goods and investment goods. Each sector consists of a unit measure of differentiated goods. In turn, each differentiated good has possibly countably many quality levels. Households rent capital to firms, which are owned by the households. Differentiated goods producers engage in research and development (R&D), which results in higher quality levels of existing products in the market. Since my model is based upon the seminal model of Klette and Kortum (2004), I briefly summarize my model, highlighting the differences with the KK model.

2.1 Households

An infinitely-lived representative household chooses time paths of consumption, capital holdings, investment in capital, and firm holdings to maximize the discounted sum of utility from consumption at time t, C_t,

$$\max \int_0^\infty \exp(-\rho t) \ln C_t dt,$$

subject to the law of motion for capital stock and a budget constraint:

$$\begin{aligned} \dot{K}_t &= X_t - \delta K_t, \\ P_{c,t}C_t + P_{x,t}X_t + \dot{A}_t &= R_t A_t + w_t L + r_t K_t - T, \end{aligned}$$

where K_t is the capital stock, X_t is investment, $P_{c,t}$ is the consumption goods price index and normalized to 1, $P_{x,t}$ is the investment goods price index, A_t is total value of the firms, R_t is the interest rate, w_t is the wage rate, L is labor supply, r_t is the rental rate of capital, and T is the lump-sum tax. Henceforth, I will drop time subscripts for notational ease.

Consumption is a CES aggregate of differentiated consumption goods:

$$C = \exp\left(\int_0^1 \ln\left(\sum_{j=0}^{J(\omega)} q^j(\omega)c^j(\omega)\right) d\omega\right), \tag{1}$$

where $q^j(\omega)$ is the quality of version j of product ω, $c^j(\omega)$ is the quantity consumed of version j of product ω, and $J(w)$ is the highest quality version of ω. As seen in Equation (1), households have perfectly substitutable preferences over the different quality-adjusted versions of each product. In equilibrium, this formulation leads to the following demand function:

$$c^j(\omega) = \begin{cases} \frac{Z}{p^j(\omega)} & \text{if } \frac{q^j(\omega)}{p^j(\omega)} \geq \frac{q^{j'}(\omega)}{p^{j'}(\omega)} \text{ for all } j' \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where $p^j(\omega)$ is the price of version j of product ω, $Z = P_c C$ is the total consumption expenditure, and I set the consumption sector basket as the numéraire, so that $P_c = \exp\left(\int_0^1 \ln\left(\frac{p(\omega)}{q(\omega)}\right) d\omega\right) = 1$.

Investment, X, is also a CES aggregate of differentiated investment goods, which are located in a different interval than consumption goods.

$$X = \exp\left(\int_0^1 \ln\left(\sum_{j=0}^{J(\omega)} q^j(\omega)x^j(\omega)\right) d\omega\right), \tag{3}$$

where $x^j(\omega)$ is the quantity invested in version j of product ω. The corresponding demand function is

$$x^j(\omega) = \begin{cases} \frac{I}{p^j(\omega)} & \text{if } \frac{q^j(\omega)}{p^j(\omega)} \geq \frac{q^{j'}(\omega)}{p^{j'}(\omega)} \text{ for all } j' \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where $I = P_x X$ is the total investment expenditure, and $P_x = \exp\left(\int_0^1 \ln \frac{p(\omega)}{q(\omega)} d\omega\right)$ is the investment price index.

2.2 Firms

A firm is defined by the set of differentiated goods it produces. Each good is produced by a unique production unit. A firm is located in single industry and can own countably many production units in that industry. It can expand the set of production units by innovating on other goods it currently does not produce. Similarly, it can lose its existing goods to other innovating firms. Furthermore, if a single good producer loses its only production unit, it exits the market. Lastly, entrepreneurs can enter the market by innovating on a good located on the unit interval.

2.3 Innovation by Incumbents

The amount of research labor a firm hires and the number of goods it produces jointly determine its Poisson innovation arrival rate, β. Innovation is not directed. The good that is innovated is randomly drawn from a uniform distribution on the unit interval of goods in the market. A firm innovates only in the sector that it currently operates in. An innovation increases the quality of the good by an industry-specific exogenous factor of $\lambda > 1$, the *quality ladder step size* which represents the innovativeness of a firm. Throughout the paper, the terms *innovativeness* or *innovative step*, will be used to signify the factor by which the quality of a product increases after a successful innovation. More innovative firms can increase the quality of a good by a larger factor. The level of innovativeness varies by sector but is invariant across firms within each sector. After a successful innovation, the innovator and the firm with the blueprints for the second highest quality version (runner-up) of the good engage in Bertrand competition; in equilibrium, the innovator charges a price equal to λ times marginal cost of production of second highest quality version of the good. Since consumers have infinitely elastic preferences over the quality adjusted varieties of a good, the innovator takes over the market. Since we identify firms with the set of differentiated goods they produce, the innovator expands by one good while the runner-up shrinks by one good.

A firm currently producing m goods and hiring l_R units of research labor innovates at a rate $\varphi(m, l_R) = \beta$, where $\varphi(\cdot)$ is a constant returns to scale production function, increasing in both arguments, and strictly concave in l_R. Like firm innovativeness, the function $\varphi(\cdot)$ varies across sectors but is invariant across firms within each sector.

Firms with experience in innovation, particularly those that retain their products despite innovation by other firms, are better at producing ideas. The number of goods in the production function is a proxy for a firm's experience in innovation.

2.4 Consumption Goods Producers

A firm in the consumption-goods sector chooses its R&D investment to maximize the value of the firm. For more detailed description of the firm problem see Appendix (B.1). Each production unit of a firm has a Cobb-Douglas production function with capital elasticity α. As a result, each production unit has the same constant marginal cost function. In equilibrium, each production unit employs the same amount of production labor, l_c, rents the same level of capital stock, k_c, and charges the same price, p_c. Moreover gross profit (before R&D expense) of each production unit is equal to

$$\pi = \left(1 - \frac{1}{\lambda_c}\right) Z. \tag{5}$$

I use the profit function in (5) to derive the firm's value function, which can be expressed either as a function of the level of research labor or, more conveniently, the level of innovation arrival rate per good. Let $m \cdot c(b_c)$ denote the level of research labor, l_R, implicitly defined by $\varphi(m, l_R) = \beta$, where $b_c \equiv \beta/m$ is the innovation rate of incumbent firms in the consumption sector. Since $\varphi(\cdot)$ is strictly increasing in l_R, and homogeneous of degree one, $c(b_c)$ is well-defined and convex in b_c. For concreteness, I assume $c(b_c) = \chi_c b_c^\gamma$, where $\chi_c > 0$ is a scale parameter.

Since profits and R&D expenditures are linear in the number of goods, the firm's value function is also linear in the number of goods.[3] The value of a production unit, denoted $\nu_c \cdot Z$, is defined by

$$R\nu_c Z = \max_{b_c \geq 0} \left\{ \left(1 - \frac{1}{\lambda_c}\right) Z - (1 - s_c^i) w c(b_c) + \nu_c \dot{Z} + b_c \nu_c Z - \tau_c n u_c Z \right\},$$

where s_c^i is the R&D subsidy rate to the incumbent firms in the consumption sector and τ_c is the innovation rate in the consumption sector. This expression equates the return on a production line (the left hand side) and the sum of gross profits, the change in the firm's value due to economic growth, and the expected value of adding a new production line, less R&D expenses and the expected value of losing the production line (the right hand side.) Simplifying the above equation yields

$$(R - g_Z + \tau_c - b_c)\nu_c = \left(1 - \frac{1}{\lambda_c}\right) - (1 - s_c^i)\chi_c b_c^\gamma \frac{w}{Z}, \tag{6}$$

where $g_Z \equiv \frac{\dot{Z}}{Z}$ is the growth rate of consumption expenditures. The first order condi-

[3]See Appendix (B.1) for more detail.

tion for innovation arrival rate per product is:

$$(1 - s_c^i) w\gamma\chi_c b_c^{\gamma-1} = \nu_c Z. \tag{7}$$

In my solution of the stationary equilibrium, the growth rate of consumption expenditures, g_Z, is constant and equal to the growth rate of wages, g_w. Appendix B.2 solves the problem and shows the equality of g_Z and g_w.

Entrepreneurs can enter the market by innovating on a product. Like incumbents, they hire research labor to develop better-quality products. An entrepreneur must hire $\xi_c(z_c, \bar{z}_c) \equiv \psi_c \chi_c z_c \bar{z}_c^{\gamma-1}$ units of labor to secure a z_c Poisson innovation rate, where the parameter $\psi_c > 0$ distinguishes the cost of innovation to incumbents from the cost to entrants, and $\bar{z}_c$ is the entry rate in the market. This reduced-form formulation of the limited availability of venture capital to entrepreneurs implies that developing a successful product requires more effort as more entrepreneurs try to enter to the market. The value of entry is therefore

$$RV_E = \max_{z_c \geq 0} \{-(1 - s_c^e) w \xi_c(z_c, \bar{z}_c) + z_c[\nu_c Z - V_E]\},$$

where s_c^e is entry subsidy rate (i.e., the entrant innovation subsidy) in the consumption sector. Free entry drives down the value of entry to zero. Hence, in equilibrium

$$(1 - s_c^e) w\psi_c \chi_n \bar{z}_c^{\gamma-1} = \nu_c Z. \tag{8}$$

2.5 Investment Goods Producers

Firms in this sector share the same Cobb-Douglas production function with capital elasticity α. The profit of a production unit, $\pi = \left(1 - \frac{1}{\lambda_x}\right) I$, is derived in a manner analogous to that of the consumption goods producers. The value of a production unit, $\nu_x I$, is

$$(R - g_I + \tau_x - b_x)\nu_x = \left(1 - \frac{1}{\lambda_x}\right) - (1 - s_x^i)\chi_x b_x^{\gamma} \frac{w}{I}, \tag{9}$$

$$(1 - s_x^i) w\gamma\chi_x b_x^{\gamma-1} = \nu_x I, \tag{10}$$

where s_x^i is the subsidy rate to R&D expenditures of incumbent firms in the investment sector, τ_x is the innovation rate in investment sector, and equation (10) is the first order condition of innovation arrival rate per product.

Entrants, on the other hand, solve

$$RV_E = \max_{z_x \geq 0} \{-(1 - s_x^e) w \xi_x(z_x, \bar{z}_x) + z_x[\nu_x I - V_E]\},$$

where s_x^e is the entry subsidy in investment sector. Free entry implies

$$(1 - s_x^e) w \psi_x \chi_x \bar{z}_x^{\gamma-1} = \nu_x I. \tag{11}$$

2.6 Equilibrium

A symmetric balanced growth path competitive equilibrium is defined by a tuple of firm decisions $\{k_{i,t}, l_{i,t}, l_{R,i}, b_i, z_i, \tau_i, c_t, x_t\}$, where $i = c, x$ represents consumption and investment sectors, a tuple of household decisions $\{c_t, x_t, C_t, X_t, K_t\}$, a tuple of prices $\{w_t, r_t, R, p_c, p_x, P_{c,t}, P_{x,t}\}$, aggregate expenditures $\{Z_t, I_t\}$, average quality levels in each sector, $\{Q_{c,t}, Q_{x,t}\}$, and value of production units per aggregate expenditure in a firm's sector, $\{\nu_c, \nu_x\}$. In equilibrium,

- $\{p_c, p_x\}$ are the Bertrand equilibrium prices of highest quality products.
- Given prices of the differentiated goods and household demand functions (2) and (4), $k_{c,t}, l_{c,t}$ and k_x, l_x solve the firm cost-minimization problems in the consumption and investment sectors.
- Given prices and nominal aggregate expenditures, $\{\nu_c, b_c, z_c\}$ solve equations (6), (7), (8), and $\{\nu_x, b_x, z_x\}$ solve equations (9), (10), (11).
- The innovation rate in sector $i = c, x$ is equal to sum of incumbent and entrant innovation rates: $\tau_i = z_i + b_i$.
- Given prices, $\{c_t, x_t, C_t, X_t, K_t\}$ are the balanced growth path values of the household optimization problem.
- The labor market clears: $l_c + l_x, +\chi_c b_c^\gamma + \chi_x b_x^\gamma + \psi_c \chi_c z_c^\gamma + \psi_x \chi_x z_x^\gamma = L$,
- The capital market clears: $K_t = k_{c,t} + k_{x,t}$,
- Nominal expenditures, $\{Z_t, I_t\}$, and wages, w_t, grow at the same rate.
- Average quality levels in the consumption and investment sectors are $Q_{c,t} = \exp\left(\int_0^1 \ln\left(q(\omega)\right) d\omega\right)$ and
  $Q_{x,t} = \exp\left(\int_0^1 \ln\left(q(\omega)\right) d\omega\right)$, where $q(\omega)$ is the highest quality level of product line ω.

This equilibrium is discussed in detail in Appendix B.2.

3 Optimality of Innovation

In keeping with Schumpeterian creative–destruction type models, the competitive equilibrium innovation rate may not be socially optimal. An innovating firm improves the quality of an existing good, destroys the profit accrued by the incumbent producer, and gains monopoly power on production of the product that it innovated. While deciding the amount of R&D to conduct, the innovating firm considers the monopoly profits that it will accrue until another firm innovates on that good and captures the product. However, the social benefit of an innovation persists forever since every innovator improves the quality upon the existing quality level. Also, innovators ignore the profit loss of the existing producer of the good. Therefore, the competitive equilibrium innovation rate is generically inefficient.

After defining the social planner problem, I discuss each externality further in Section 3.2. The allocation of innovative resources across industries depends both on the locations of industries in the supply chain and the differences in the innovation processes of industries. Sections 3.3 and 3.4 analyze the role of the former factor on the allocation of innovative resources across industries by focusing on a special case of the model in which industries share identical innovation processes (i.e., when $\lambda_c = \lambda_x$, $\chi_c = \chi_x$, and $\psi_c = \psi_x$).

In order to identify how the externalities affect the economy, I define and solve the social planner's problem. Then, I compare the competitive equilibrium first order conditions with the social planner first order conditions, and discuss the distortions caused by externalities.

3.1 Social Optimum

The social planner maximizes the discounted sum of utility from consumption, C_t:

$$\max \int_0^\infty e^{-\rho t} \ln(C_t) dt \quad \text{subject to}$$

- the resource constraint on aggregate consumption: $C_t = K_{c,t}^{\alpha} L_{c,t}^{1-\alpha} Q_{c,t}$, where $K_{c,t}$ and $L_{c,t}$ are the capital stock and the production labor allocated to the consumption goods sector, and $Q_{c,t}$ is the average quality in the consumption sector,
- the resource constraint on the allocation of capital across sectors : $K_{c,t} + K_{x,t} = K_t$, where $K_{x,t}$ is the capital stock allocated to the investment sector and K_t is the aggregate capital stock at time t,
- the resource constraint on the allocation of labor across production and R&D activity: $L_{c,t} + L_{x,t} + \psi_c \chi_c z_{c,t}^{\gamma} + \chi_c b_{c,t}^{\gamma} + \psi_x \chi_x z_{x,t}^{\gamma} + \chi_x b_{x,t}^{\gamma} \leq 1$, where $L_{x,t}$ is the production labor allocated to the investment sector, $z_{i,t}$ is the entry rate and $b_{i,t}$ is the incumbent firm innovation rate for $i = c, x$,
- the innovation rate in a sector, $\tau_{i,t}$ for $i = c, x$, being equal to sum of innovation rates of entrants and incumbent firms: $z_{c,t} + b_{c,t} = \tau_{c,t}$ and $z_{x,t} + b_{x,t} = \tau_{x,t}$,
- the law of motion for capital stock: $\dot{K}_t = K_{x,t}^{\alpha} L_{x,t}^{1-\alpha} Q_{x,t} - \delta K_t$, where $Q_{x,t}$ is the average quality in the investment goods sector,
- the average quality levels in each sector, $Q_{c,t} = \exp\left(\int_0^1 \ln\left(q_t(\omega)\right) d\omega\right)$, $Q_{x,t} = \exp\left(\int_0^1 \ln\left(q_t(\omega)\right) d\omega\right)$,
- and laws of motion for the average quality (or technology index) of the consumption sector, $\frac{\dot{Q}_{c,t}}{Q_{c,t}} = \tau_{c,t} \log \lambda_c$, and the investment sector, $\frac{\dot{Q}_{x,t}}{Q_{x,t}} = \tau_{x,t} \log \lambda_x$.

The social planner problem (SP) can be divided into two parts: 1) a static problem that allocates a given level of total innovation in a sector to entrants and incumbents, and 2) a dynamic problem that determines the time paths of labor, capital and innovation.

In the static problem, the social planner chooses entry and incumbent innovation rates in each sector to minimize the research cost of a fixed level of aggregate innovation in that sector:

$$\min_{z_i,b_i} \psi_i \chi_i z_i^\gamma + \chi_i b_i^\gamma \quad \text{subject to} \quad z_i + b_i = \tau_i,$$

where z_i is the entry rate, b_i is the innovation rate by incumbents, τ_i is the aggregate innovation rate in sector $i = c, x$.[4] Note that the social planner takes into account the externality created by entrants on each other. The resulting cost function (in labor units) for a sector is

$$C_i(\tau_i) = \frac{\psi_i \chi_i \tau_i^\gamma}{\left(1 + \psi_i^{1/(\gamma-1)}\right)^{\gamma-1}}. \tag{12}$$

The economy-wide research cost function is the sum of innovation costs across sectors,

$$C(\tau_c, \tau_x) = \frac{\psi_c \chi_c \tau_c^\gamma}{\left(1 + \psi_c^{1/(\gamma-1)}\right)^{\gamma-1}} + \frac{\psi_x \chi_x \tau_x^\gamma}{\left(1 + \psi_x^{1/(\gamma-1)}\right)^{\gamma-1}}.$$

Using the research labor cost function from the static problem, the social planner then maximizes the discounted sum of utility from consumption,

$$\max \int_0^\infty e^{-\rho t} \ln(K_{c,t}^\alpha L_{c,t}^{1-\alpha} Q_{c,t}) dt,$$

subject to constraints stated above, with the labor constraint rewritten as $L_{c,t} + L_{x,t} + C(\tau_{c,t}, \tau_{x,t}) \leq 1$.

3.2 Distortions

Innovative activity leads to various distortions in the market equilibrium conditions relative to the social optimum. First, an improvement in the quality level of a good gives market power to the innovator, i.e. she can charge a markup over the marginal cost of production. Second, quality improvements occur over existing innovations ('standing on the shoulders of giants'). Hence, an innovation increases the quality level of a good forever, but the innovator gets the benefit for a limited time, until the next innovation on the good. Third, innovation destroys the profit accruing to the incumbent ('business stealing'). Fourth, the cost of entry into the market by an entrepreneur increases with the measure of total innovative activity by entrants. The first order condition for innovation rates (denoted with ' ˆ ') in competitive equilibrium, and first order condition for social planner innovation (denoted with ' *

[4]Note that time index is dropped for the sake of simplicity.

') are as follows:

$$c'(\hat{b})w = \frac{1(\pi - c(\hat{b})w)}{\rho + \tau - b}, \tag{13}$$

$$c'(b^*)F_L(K, L, Q) = \frac{\ln(\lambda)F(K, L, Q)}{\rho}, \tag{14}$$

where $c(\cdot)$ is the R&D cost function in the competitive equilibrium and $F(\cdot)$ is sector-level production function.[5] In equation (14), I rely on the envelope condition: $C'(\tau_i) = c'(b_i)$ for $i = c, x$ in the social planner's problem. Although I drop sector subscripts for notational ease, equations (13) and (14) hold for each sector. Following Aghion and Howitt (1992), I compare equations (13) (competitive equilibrium first order condition) and (14) (social planner F.O.C.) to understand the effect of these distortions on innovation levels.

These F.O.C.s equate the marginal cost of innovation to the discounted benefit of innovation. The marginal cost of innovation in the competitive equilibrium is $c'(b)w$, while the marginal cost in the SP problem is $c'(b)F_L(K, L, Q)$. Since firms have monopoly power, the marginal product of labor may differ from the wage rate. This *monopoly-distortion effect* causes the competitive equilibrium innovation level to exceed the SP level (Aghion and Howitt (1992)). Second, the private flow benefit of innovation is the monopoly profit minus research cost, $\pi - c(b)w$, whereas the social benefit is total output $F(K, L, Q)$, so that b^* exceeds $\hat{b}$ (*appropriability*). Third, as a result of innovation, the monopolist takes over the market for the good, ignoring the loss incurred by the incumbent, hence the '1' factor on π. In contrast, the social planner considers the change in utility as a result of collective innovation, hence the $\log \lambda$ factor on $F(\cdot)$. This *business-stealing effect* leads to a higher level of private innovation. Fourth, the private innovator accrues the benefits as long as she has monopoly power over the good. Therefore, she discounts the profits at a rate $\rho + \tau - b$. In contrast, the benefits of innovation accrue to society forever, since the quality increase persists through time. This *inter-temporal spillover effect* yields higher social planner innovation levels.

The Euler equations in the market economy and social planner problem are

$$\frac{1}{\lambda_x}\alpha \hat{K}_x^{\alpha-1}\hat{L}_x^{1-\alpha}Q_x - \delta - \rho = \frac{1}{1-\alpha}\hat{\tau}_x \ln \lambda_x, \tag{15}$$

$$\alpha {K^*}_x^{\alpha-1}{L^*}_x^{1-\alpha}Q_x - \delta - \rho = \frac{1}{1-\alpha}\tau_x^* \ln \lambda_x. \tag{16}$$

Monopoly pricing distorts the price of the investment good, leading to differences in these Euler equations. Specifically, the marginal product of capital in the investment goods sector in competitive equilibrium is shrunk by a factor of $1/\lambda_x < 1$ relative to the social planner problem, resulting in lower private capital in the competitive

[5]The R&D cost function is $c_i(b_i) = \chi_i b_i^\gamma$, $i = c, x$.

equilibrium. However, innovation in investment goods also affects the change in the relative price of investment goods. The higher the innovation, the greater the decline in the price of investment goods. The greater pace of decline in the price of investment goods makes the acquisition of capital in initial periods more costly. Hence, regimes that have a higher innovation in investment goods have lower level of capital.

While not the focus of this paper, the entry process creates another externality: entrants do not internalize the extra entry cost they impose on other entrants. Equations (17) and (18) show the competitive equilibrium and social planner first order conditions for the allocation of innovation between entrants and incumbents. While the social planner allocation equates the marginal cost of entry and the marginal cost of incumbent innovation, these quantities differ in the competitive equilibrium, leading to a more than optimal entry rate.

$$\psi\chi\hat{z}^{\gamma-1} = \gamma\psi\chi\hat{b}^{\gamma-1} \tag{17}$$

$$\gamma\psi\chi z^{*\gamma-1} = \gamma\psi\chi b^{*\gamma-1} \tag{18}$$

Lastly, since capital and labor markets are competitive, the only distortion in the factor demand equations comes from monopoly pricing of the goods. Equating relative factor prices across sectors yields the undistorted capital labor ratios. Equation (19) is identical in market equilibrium and in the social planner allocation:

$$\frac{1-\alpha}{\alpha}\frac{K_x}{L_x} = \frac{1-\alpha}{\alpha}\frac{K_c}{L_c}. \tag{19}$$

3.3 Optimal Allocation of Innovative Resources Across Sectors

Section 3.2 characterized the distortions in the economy that yield different sector-specific innovation rates in the competitive equilibrium and social planner problems. In this section, I characterize the optimal allocation of innovative resources across sectors.

When assigning innovation rates to industries, the social planner considers, among other factors, (i) any differences in the innovation process (as captured by innovativeness, λ_i, and the costliness of innovation, χ_i) across sectors; and (ii) the location of industries in the supply chain. This section focuses on the latter factor, analyzing the effect of an industry's location in the supply chain on the optimal relative innovation rate in that industry. To isolate the impact of the location of an industry on the innovation rate, I consider a special case of the model where investment goods producers use only labor in production, that is, $X_t = L_{x,t}Q_{x,t}$. In this formulation, the investment sector is a clear upstream industry. Substituting the production functions into equation (14) at the balanced growth path yields

$$C_c'(\tau_c) = \frac{\ln\lambda_c L_c/(1-\alpha)}{\rho}, \quad \text{and} \quad C_x'(\tau_x) = \frac{\ln\lambda_x L_x}{\rho}.$$

Similarly, equation (16) is rewritten as

$$-\delta + \frac{\alpha}{1-\alpha}\frac{L_c}{K/Q_x} = \rho + \tau_x \ln \lambda_x.$$

The equality of the growth rates of investment and capital stock along the balanced growth path (BGP) implies

$$\frac{L_x}{K/Q_x} - \delta = \tau_x \ln \lambda_x.$$

Rearranging these first order conditions at the BGP leads to the optimality condition,

$$\frac{C_c'(\tau_c)}{C_x'(\tau_x)} = \frac{1}{\alpha}\frac{\ln \lambda_c}{\ln \lambda_x}\frac{\rho + \delta + \tau_x \ln \lambda_x}{\delta + \tau_x \ln \lambda_x}, \tag{20}$$

and the labor resource constraint,

$$1 = \frac{\rho C_c'(\tau_c)(1-\alpha)}{\ln \lambda_c} + \frac{\rho C_x'(\tau_x)}{\ln \lambda_x} + C_c(\tau_c) + C_x(\tau_x). \tag{21}$$

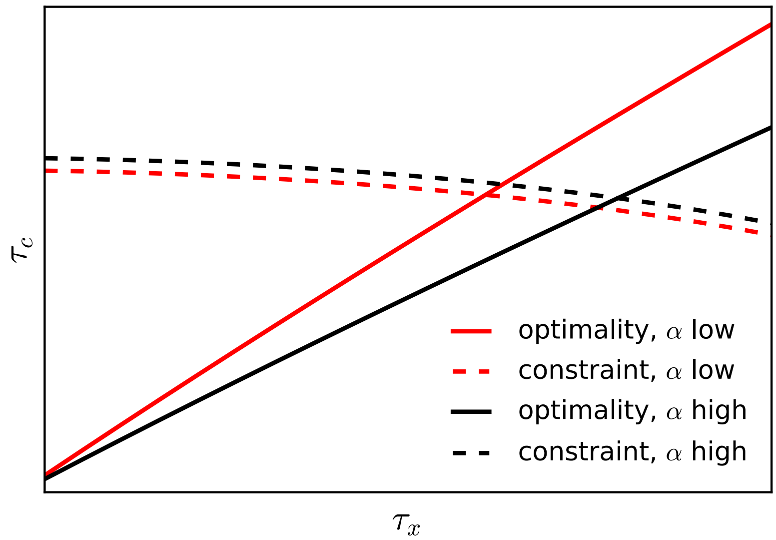
Equation (20) shows that the allocation of innovative resources across sectors depends on the relative *influence* of sectors, $1/\alpha$, which in turn depends on the location of industries in the supply chain. Here, the *influence* of a sector is defined as the elasticity of TFP with respect to the sector's productivity (Acemoglu et al. (2012), Bigio and Lao (2020)). As α increases, the investment sector becomes more influential, therefore the social planner raises the innovation rate in this sector. Figure 1 depicts equations (20) and (21). The intersection of the optimality (20) and constraint (21) equations gives the socially optimal innovation rates at the balanced growth path. Figure 1 illustrates the optimal innovation rates at two different values of α: low and high. The high-α optimality curve lies to the right of the low-α optimality curve; for a given consumption sector innovation rate, τ_c, the optimal investment sector innovation rate, τ_x, increases with the investment sector's influence. Further, equation (21) shows that an increase in the investment sector's influence frees up labor from production for the social planner to allocate to innovation; the high-α constraint curve lies above the low-α constraint curve. In summary, a more influential investment sector implies an unambiguously higher socially optimal innovation rate in the investment sector, but may result in higher or lower consumption sector innovation rate depending on the relative shifts in the optimality and constraint conditions.[6]

3.4 Misallocation of Innovative Resources Across Sectors

Section 3.3 characterized the optimal allocation of innovative resources across sectors. In the current section, I describe the market allocation of innovative resources, contrasting it with the socially optimal allocation. As in the previous section, my focus is

[6] Please see Appendix H for further analysis on this point in a multi-sector version of the model.

Figure 1: Social planner allocation of innovative resources across sectors



Notes: The solid lines represent the optimality condition in equation (20) for low and high values of α, whereas the dashed lines represent the constraint equation (21). The intersection of the optimality and constraint curves yields the social planner's choice of innovation rates in the two industries.

on understanding the differences in sectoral innovation rates stemming from the location of industries in the supply chain. Therefore, I continue working with the special case of the model in which labor is the only factor of production in the investment sector. I further assume that the industries share the same innovation functions: they have identical innovative steps ($\lambda_x = \lambda_c = \lambda$) and cost functions ($\psi_x = \psi_c = \psi$ and $\chi_x = \chi_c = \chi$). This special case rules out differences in innovation rates stemming from functional form differences and highlights the role of the location of an industry in determining its innovation rates. In this subsection, I describe the effects of each of the distortions explained in Section 3.2 on the allocation of innovative resources across industries.

Consider the competitive equilibrium first-order condition for the innovation rate in equation (13). Assuming entry is subsidized such that a given level of innovation in an industry is optimally allocated to entrants and incumbents, the ratio of innovation rates across industries in to the competitive equilibrium is

$$\frac{C'(\tau_c)(\rho + z_c) + c(b_c)}{C'(\tau_x)(\rho + z_x) + c(b_x)} = \frac{\pi_x}{\pi_c} = \frac{1}{\alpha}\lambda\frac{\rho + \delta + \tau_x \ln\lambda}{\delta + \tau_x \ln\lambda}, \tag{22}$$

where the second equality follows from the F.O.C.s at the BGP.[7,8] Comparing equation

[7]Entry subsidy equal $1 - (1 - s_i)\gamma$ achieves conditional optimality, where s_i is the incumbent R&D subsidy.

[8]Equation (22) would be as follows if I had not focused on the special case with identical innovation functions:

$$\frac{C_c'(\tau_c)(\rho + \tau_c - b_c) + c_c(b_c)}{C_x'(\tau_x)(\rho + \tau_x - b_x) + c_x(b_x)} = \frac{\pi_x}{\pi_c} = \frac{1}{\alpha}\frac{\lambda_c - 1}{\lambda_x - 1}\lambda_x\frac{\rho + \delta + \tau_x \ln\lambda_x}{\delta + \tau_x \ln\lambda_x}.$$

(22) to its socially-optimal counterpart in equation (20) highlights three sources of misallocation of innovative resources across sectors.

First, the quality ladder step size, λ, appears on the right hand side of equation (22). Market power in the investment sector distorts the equilibrium allocation of production, so that the profit ratio differs from the production ratio.

Second, the R&D cost functions, $c(b_c)$ and $c(b_x)$, appear on the left hand side of equation (22) but not in equation (20) because the flow return to innovation in the competitive equilibrium is gross profits, net profits minus R&D costs (as incumbents firms keep innovating). Future R&D flow costs drive a wedge between the ratio of the flow return of innovation and the ratio of marginal social benefit of innovation. The first two sources of misallocation are related to the appropriability effect, which arises when net profits, $\pi - c(b)w$ differ from production, $F(K, L, Q)$. Therefore, the ratio of net industry profits may differ from the ratio of industry production because gross profit ratios may differ from the production ratios and commitment to future R&D reduces inventors' net profits.

Third, the entry rates z_c and z_x appear on the left hand side of equation (22) but not in equation (20) because inventors do not internalize benefits of their innovation to future producers (the inter-temporal spillover effect) resulting in the misallocation of innovative resources across industries. The monopolist distortion of wages and the business stealing effect, on the other hand, do not contribute to the misallocation of innovative resources across sectors as both industries have the same wage and the business stealing effect is equal across industries when innovative steps do not differ across industries (i.e., when $\lambda_c = \lambda_x$).

I now analyze the impact of these distortions on the relative innovation rate in the consumption sector, τ_c/τ_x, by adding one distortion at a time to the social planner allocation in the special case of the model described at the beginning of the current section. The first distortion, monopoly power of the investment sector, distorts innovation rates toward the consumption sector. To understand the intuition, suppose that the influence ratio is equal to the gross profit ratio as in the competitive equilibrium, then equation (20) becomes

$$\frac{C'(\tau_c)}{C'(\tau_x)} = \frac{\lambda}{\alpha} \frac{\rho + \delta + \tau_x \ln \lambda}{\delta + \tau_x \ln \lambda}.$$

Since $\lambda > 1$ and $C'(\cdot)$ is an increasing function, introducing monopoly power shifts up the optimality curve in Figure 1 with no impact on the constraint curve, resulting in a lower innovation rate in the investment sector and a higher innovation rate in the consumption sector. Therefore, $\tau_c/\tau_x > \tau_c^*/\tau_x^*$, where τ_c^*/τ_x^* is the socially optimal relative innovation rate. Intuitively, monopoly power distorts profits away from the upstream (investment) industry toward the downstream (consumption) industry, thereby increasing the innovation rate in the downstream industry and reducing the innovation rate in the upstream industry.

Incorporating the inter-temporal spillover effect (the third distortion) in the social planner problem distorts relative innovation rates towards the industry with lower innovation rates. Embedding inter-temporal spillovers effect into equation (20) yields:

$$\frac{C'(\tau_c)(\rho+z_c)}{C'(\tau_x)(\rho+z_x)} = \frac{1}{\alpha}\frac{\rho+\delta+\tau_x \ln\lambda_x}{\delta+\tau_x\ln\lambda_x}.$$

Similarly, equation (21) becomes

$$1 = \frac{(\rho+z_c)C'(\tau_c)(1-\alpha)}{\ln\lambda} + \frac{(\rho+z_x)C'(\tau_x)}{\ln\lambda} + C(\tau_c) + C(\tau_x).$$

Since the entry rate is proportional to the industry innovation rate,[9] the inter-temporal spillover effect pushes the optimality condition in Figure 1 in the direction of the industry with low innovation rate. This shifts the optimality curve downward in the special case of the model (described at the beginning of the current section) because the investment sector has a lower innovation rate. Similarly, the constraint curve also shifts down. Overall, the consumption sector innovation rate, τ_c, falls, whereas the impact of adding the inter-temporal spillover effect into the social planner problem on the investment-sector innovation rate, τ_x, is ambiguous. Therefore, the direction of the change in τ_c/τ_x is also ambiguous. However, all the numerical comparative statics I conducted with the model parameters show that τ_c/τ_x goes down. Incorporating the inter-temporal spillover effect into the social planner problem pushes innovation away from downstream industry towards upstream industry.

Adding net profit considerations (the second distortion) into the social planner problem shifts innovation towards the industry with lower innovation rate. Adding the net profit motive to the social planner problem transforms equation (20) into

$$\frac{C'(\tau_c)\rho + c(b_c)}{C'(\tau_x)\rho + c(b_x)} = \frac{1}{\alpha}\frac{\rho+\delta+\tau_x \ln\lambda_x}{\delta+\tau_x\ln\lambda_x}.$$

Similarly, equation (21) becomes:

$$1 = \frac{[\rho C'(\tau_c) + c(b_c)](1-\alpha)}{\ln\lambda} + \frac{\rho C'(\tau_x) + c(b_x)}{\ln\lambda} + C(\tau_c) + C(\tau_x).$$

With the current parameterization of the model, the cost of innovation in industry i is equal to $C(\tau_i) = A\tau_i^\gamma$, where A is a reduced form parameter.[10] Maintaining the assumption of identical innovation functions, and noting that the social planner equalizes the incumbent share of total innovation, b/τ, across sectors yields

$$\frac{\tau_c^{\gamma-1}(\gamma A\rho + \chi a^\gamma \tau_c)}{\tau_x^{\gamma-1}(\gamma A\rho + \chi a^\gamma \tau_x)} = \frac{1}{\alpha}\frac{\rho+\delta+\tau_x \ln\lambda_x}{\delta+\tau_x\ln\lambda_x},$$

for some reduced form parameter a.[11]

[9] The social planner allocates a constant fraction, $z_i = \tau_i(1+\psi^{1/(\gamma-1)})^{-1}$, of total innovation to entrants in industry $i = c, x$.

[10] Using the cost of innovation functions of the entrants and incumbents, the cost of total innovation in industry i can be written as $C(\tau_i) = A\tau_i^\gamma$ where $A \equiv \chi\left(\frac{\psi+\psi^{\gamma/(\gamma-1)}}{(1+\psi^{1/(\gamma-1)})^\gamma}\right)$.

[11] In equalizing the incumbent share of total innovation across sectors, the social planner allocates a constant

When the innovation rate in the consumption sector exceeds that in the investment sector ($\tau_c > \tau_x$), the left hand side of the optimality condition is multiplied by $(\gamma A\rho + \chi a^{\gamma}\tau_c)/(\gamma A\rho + \chi a^{\gamma}\tau_x) > 1$. Therefore, the optimality condition in Figure 1 shifts downward, whereas the constraint condition shifts inward, lowering τ_c, and having an ambiguous effect on τ_x. However, the numerical comparative statics suggest that the relative innovation rate in the consumption sector, τ_c/τ_x, falls upon introducing this particular distortion into the social planner problem. In summary, the net profit consideration shifts innovation away from the downstream industry into the upstream industry.

Table 1 summarizes the results of this section. Section 3.3 highlights the effects of various distortions on the innovation rate of an industry in a partial equilibrium fashion. The current section characterizes the effects of such distortions on the allocation of innovation across industries in a special case of the model, where innovation functions of the industries are identical. Exercises with this special case highlight the disproportionate effects of distortions on industries at different locations of the supply chain. First, the investment sector's monopoly power distorts the economy by reducing the production of the upstream industry and hence reduces marginal gain of innovation in the upstream industry, shifting innovation towards the downstream industry; $\tau_c/\tau_x > \tau_c^*/\tau_x^*$. Second, the inter-temporal spillover effect pushes innovation toward the industry with the lower innovation rate. When inventors expect that future inventors can take over their market, they reduce their innovation efforts. Similarly, if the social planner takes this motive into account, she shifts innovation from the high innovation industry to the low innovation industry, which, in the special case of my model, is the upstream investment sector. Lastly, the net profit consideration affects the allocation of innovation across industries in a manner similar to the inter-temporal spillover effect, moving innovation away from the consumption sector into the investment sector.

To summarize the theoretical results, in Section 3.2, I identify the inefficiencies in the innovation process using the terminology developed by Aghion and Howitt (1992) and Grossman and Helpman (1991). In Sections 3.3 and 3.4, I contribute to the literature by analyzing the optimal allocation of innovative resources across industries and the misallocation of resources in the market economy. The optimal allocation of resources across industries depends on the location of the industry in the supply chain and differences in the innovation functions of the industries.

In Sections 3.3 and 3.4, by focusing on a special case of my model, I analyze the effects of various distortions on the innovation rates of industries in different parts of the supply chain. When industries have identical innovation functions, the social planner allocates more resources to innovation in the downstream industry. Further, as the *influence* of the upstream industry increases, so does the innovative resources allocated to the upstream industry in the social planner problem. Later, I show

fraction of total innovation in industry i to incumbents, $b_i = a\tau_i$, where $a \equiv \frac{\psi^{1/(\gamma-1)}}{1+\psi^{1/(\gamma-1)}}$.

Table 1: Distortions

Distortion	Condition	Effect
Monopoly power of the investment sector	–	$\tau_c/\tau_x > \tau_c^*/\tau_x^*$
Inter-temporal spillover effect	$\tau_c > \tau_x$	$\tau_c/\tau_x < \tau_c^*/\tau_x^*$
Net profit consideration	$\tau_c > \tau_x$	$\tau_c/\tau_x < \tau_c^*/\tau_x^*$
Business stealing effect	$\lambda_c = \lambda_x$	No effect
Monopoly distortion of wages	–	No effect

Notes: This table summarizes the arguments made in Section 3.4 in the special case of the model where (i) capital is used only for consumption goods production (not for investment goods production); and (ii) the innovation costs functions and innovativeness are identical across industries. The rows correspond to the distortions affecting the allocation of innovation. The "Condition" column shows the setting under which the distortion in question yields the effect described in the "Effect" column.

that the distortions identified in Section 3.2 also effect the allocation of innovative resources across industries. Monopoly power shifts innovative resources towards the consumption (downstream) sector, and the inter-temporal spillover effect and net-profit consideration shift resources towards the industry with lower innovation rate in the SP problem (the investment sector in the special case of the model). The business stealing effect, in the special case of the model with identical innovation functions, does not alter the allocation of resources between industries. Similarly, monopoly distortion on wages, effects each sector equally and has no effect on the allocation of innovative resources across industries.

4 Calibration

I calibrate the balanced growth path (BGP) of my model to averages of data on non-financial corporate sector from 1987 to 2017, subject to the availability of data. A unit length of time in the model is considered as a year in the data. Growth rate of output is targeted to match the average growth rate of real gross value added of non-financial corporate sector, less net taxes on production and imports, less investment in intellectual property products per worker, so that $g_Y = 0.015$. Similarly, labor's share of income is calculated as compensation of employees over gross value added, less net taxes on production and imports, less investment in intellectual property products per worker.[12] The discount rate is targeted to have a 0.97 annual discount factor, which

[12]Components of value added data are obtained from the Bureau of Economic Analysis (BEA). Intellectual property data is obtained from the Integrated Macroeconomic Accounts for the United States, published by the Board of Governors of the Federal Reserve System (US).

implies a BGP interest rate, $R = 4.5\%$, slightly higher than the estimates in Hall (2003) and McGrattan and Prescott (2005). The depreciation rate, δ, is calibrated to have a 5% annual deprecation rate.[13]

To calibrate the curvature of the R&D cost function, γ, I target the price elasticity of R&D with respect to its user cost, estimated by Bloom et al. (2002). They estimate both short-run and long-run elasticities of R&D with respect to its user cost. The short–run elasticity, defined as the immediate effect of user cost changes, is estimated as 0.35, which corresponds to $\gamma = 3.85$ in my model. The long-run elasticity, the sum of R&D changes in all subsequent periods,[14] is approximately 1, which corresponds to a $\gamma = 2$ in my model. However, neither of these estimates correspond exactly to my model, where firms make R&D decisions in each period and reap the benefits of R&D immediately. In reality, firms commit to R&D for certain periods of time, but not indefinitely. Therefore, I set $\gamma = 2.5$, approximately midway between the short-run and long-run elasticities. The R&D subsidy rates for incumbent firms in the two sectors are set to 0.1 to match the percentage of business R&D financed by government.[15] Since the focus of this paper is on the inefficiencies of the total innovation rate in a sector, I remove the entry-related inefficiency by subsidizing/taxing entry accordingly, $s_{e,j} = 1 - (1 - s_{i,j})\gamma$, for $j = c, x$.

Table 2 reports the parameters that are calibrated independently from the data or taken from other papers.

Table 2: Externally calibrated parameters

	Parameter	Value
Depreciation Rate	δ	0.05
Discount Rate	ρ	0.03
Curvature of R&D cost function	γ	2.50
R&D subsidy, consumption incumbents	s_c^i	0.10
R&D subsidy, investment incumbents	s_x^i	0.10
R&D subsidy, consumption entrants	s_c^e	-1.25
R&D subsidy, investment entrants	s_x^e	-1.25

Notes: R&D subsidies to entrants are chosen to eliminate the congestion externality that entrants impose on each other because this externality is not a focus of the paper.

Other parameters of the model are calibrated by using the implications of the model on firm entry and expansion rates. The innovation rate by entrants corresponds to the firm entry rate — the measure of entering production units over the total measure of production units in the sector. The innovation rate by incumbents corresponds to firm expansion rates — the measure of production units captured by incumbent firms over the total measure of production units in the sector. Further, since each production

[13] KLEMS data on U.S. See Jäger (2017) for details.

[14] Firms' R&D expenditures are highly persistent, so that a change in user cost at the current period affects R&D in all subsequent periods.

[15] OECD data on gross domestic expenditure on R&D by sector of performance and source of funds.

unit in a sector employs the same amount of labor, the innovation rate by entrants is equal to the number of jobs created by entering firms over the total employment in that industry (job creation rate by birth). Similarly, the innovation rate by incumbents is equal to the ratio of the number of jobs created by expanding firms to the total employment in that industry, the *job creation rate by expansion*.[16] To match the data with my model, I make the identifying assumption that an establishment in the data corresponds to a firm in the model.

I then link observed industries to final goods industries in my model. Each industry in the data produces output that is used as a final good consumption, final good investment, and intermediate input to other industries. Hence, there is no one-to-one link between industries in the data and final good industries in my model. To establish such a link, I first calculate – using the BEA Input-Output tables – the amount of labor required from each industry to produce one unit of each final consumption and investment good. Second, using these industry labor contents of final goods production, I construct entry and expansion rates in the final goods industries as weighted averages of industry entry and expansion rates in the data. Appendix A details the procedure behind the construction of targets for final goods industries. The first four rows of Table 3 show the targeted innovation rates among entrants and incumbents in each sector.

Table 3: Targets

	Variable	Data	Model
Entrant innovation rate, consumption	z_c	0.052	0.052
Entrant innovation rate, investment	z_x	0.047	0.047
Incumbent innovation rate, consumption	b_c	0.097	0.097
Incumbent innovation rate, investment	b_x	0.099	0.099
GDP per worker growth rate	g_Y	0.015	0.015
Growth rate of investment good prices relative to consumption good prices	g_{P_x}	-0.028	-0.028
Labor's share of income		0.714	0.714

Notes: This table reports the moments targeted during estimation. The "Data" column represents values of these moments in the data, and "Model" column shows the same moments in the balanced growth path of the model.

The model also has implications on the growth rate of the the relative price of investment goods, g_{P_x}. The growth rate of the price of quality adjusted investment goods is approximately -2.8% (Gordon (1990), Cummins and Violante (2002) and DiCecio (2009)). Technological progress in each industry contributes to the growth rate of consumption, g_C, (equal to the growth rate of GDP per worker, g_Y, in equation 23), whereas the growth rate of the relative price of investment goods depends on the difference of technological progress in each sector (equation 24):

[16] Business Dynamics Statistics (BDS) provides job creation rates by entering establishments and job creation rates by expansion of establishments for major SIC industries.

$$g_Y = \tau_c \ln \lambda_c + \frac{\alpha}{1-\alpha} \tau_x \ln \lambda_x, \tag{23}$$

$$g_{P_x} = \tau_c \ln \lambda_c - \tau_x \ln \lambda_x. \tag{24}$$

Therefore, the innovativeness in the two sectors, λ_c and λ_x, are identified using Equations (23) and (24) and target rates for the growth rate of GDP per worker, the change in the relative price of investment goods, and the innovation rates in each industry.

The other parameters of the model, in Table 4, are calibrated to match the model moments and the target moments in the data. In the model, labor's share of income is equal to the sum of payments to production labor and the R&D labor of incumbents over GDP.[17] The relative costs of entry ψ_x, ψ_c, in each sector are identified by job creation by birth over job creation by expansion rate in these industries. Overall, the results of the calibration exercise indicate that: (1) the quality ladder step size of investment goods is higher than that of consumption goods ($\lambda_x > \lambda_c$); (2) innovation in the investment goods sector is more costly ($\chi_x > \chi_c$); (3) innovation is costlier for entrants ($\psi_x, \psi_c > 1$); and (4) entry is more costly in the investment sector ($\psi_x > \psi_c$).

Table 4: Internally calibrated parameters

	Parameter	Value
Quality ladder step size, investment	λ_x	1.24
Quality ladder step size, consumption	λ_c	1.02
R&D cost function parameter, investment	χ_x	6.30
R&D cost function parameter, consumption	χ_c	3.03
Entry cost function parameter, investment	ψ_x	3.05
Entry cost function parameter, consumption	ψ_c	2.56
Elasticity of output w.r.t capital	α	0.27

Notes: Table of internally calibrated parameters that are estimated by minimizing the distance between model moments and data moments shown in Table 3.

5 Numerical Analysis

Of the distortions identified in Section 3.2, appropriability and inter–temporal spillover effects cause the economy to under-invest in innovation whereas business stealing and monopoly distortion cause the economy to over-invest in innovation; whether the economy under- or over-invest in innovation depends on the parameters of the model. As shown in Table 5, the competitive equilibrium of the model with the calibrated parameters suggests that the economy under-invests in innovation. Column 1 shows the competitive equilibrium consumption growth rate, innovation rates in sectors, and the discounted capital stock, $\tilde{K} = \frac{K}{Q_x^{1/1-\alpha}}$, defined as the capital stock level at

[17] Note that intellectual property production (R&D) is not included in GDP in the model.

the steady state of the economy, with variables discounted appropriately using the technology indices. Column 2 shows the social planner values. The socially optimal innovation rate is 13 percentage points higher than the competitive equilibrium rate in the consumption sector and 17 percentage points higher in the investment sector. These higher innovation rates result in the economy growing 1.7 percentage point faster under the social planner. In a similar exercise, using a Schumpeterian creative destruction model whose parameters are estimated using Danish firm level data, Lentz and Mortensen (2015) find that the optimal growth rate is twice as large as the competitive equilibrium growth rate.[18]

Later, I discuss the welfare implications of R&D subsidies that push the economy towards the social planner allocations. The change in the consumption growth rate will play an important role in generating welfare gains. The other important factor that needs to be analyzed is capital stock. Removing the monopoly distortion in the capital Euler equation (15) leads to higher capital accumulation. However, a higher user cost of capital, resulting from higher innovation in investment goods, would result in lower accumulation of capital. In the quantitative exercise, the latter force dominates and steady state capital stock of the social planner is lower than capital stock in the market economy. A transition of the economy from competitive equilibrium innovation levels to social planner innovation levels would cause a decline in the labor allocated to consumption good production. On the other hand, if the amount of capital invested also decreases then some of the labor allocated to investment good production can be used in consumption good production or research. This will create extra welfare gain in the economy.

Table 5: Competitive Equilibrium vs Social Planner

	CE	SP		
			CE τ_c	CE τ_x
g_C	0.015	0.032	0.030	0.019
τ_c	0.149	0.277	0.149	0.319
τ_x	0.146	0.321	0.331	0.146
$\tilde{K}$	2.031	1.293	1.320	2.511

Notes: Column 1 (CE) shows the competitive equilibrium values, column 2 (SP) the social planner values, column 3 (CE τ_c) the social planner values when restricted to the competitive equilibrium innovation rate in consumption sector, and column 4 (CE τ_x) when the social planner is restricted to the competitive equilibrium innovation rate in investment sector.

To gauge the relative importance of innovations in the two sectors, I conduct the following exercises which are shown in columns 3 (CE τ_c) and 4 (CE τ_x) of Table 5. In the exercise depicted in column 3, I solve the social planner problem while constraining

[18] They consider a model where firms in a sector have different innovativeness which evolve according to a Markov Process.

the innovation rate in the consumption sector to the competitive equilibrium value. In this setting, the social planner allocates more labor to innovative activity in the investment sector and reaches a consumption growth rate close to the unconstrained problem rate. However, in a similar exercise where the investment sector innovation rate is constrained to its competitive equilibrium level, the consumption growth rate is 1.3 percentage points lower than the social planner rate (reported in column 4). Although the social planner increases the innovation rate in the consumption sector, this adjustment does not offset the decline in investment sector innovation. These exercises indicate that the socially optimum growth rate is significantly higher than the growth rate in the market economy. Further, it is the investment sector under-innovation that leads to a large gap between the competitive equilibrium and socially optimal growth rates.

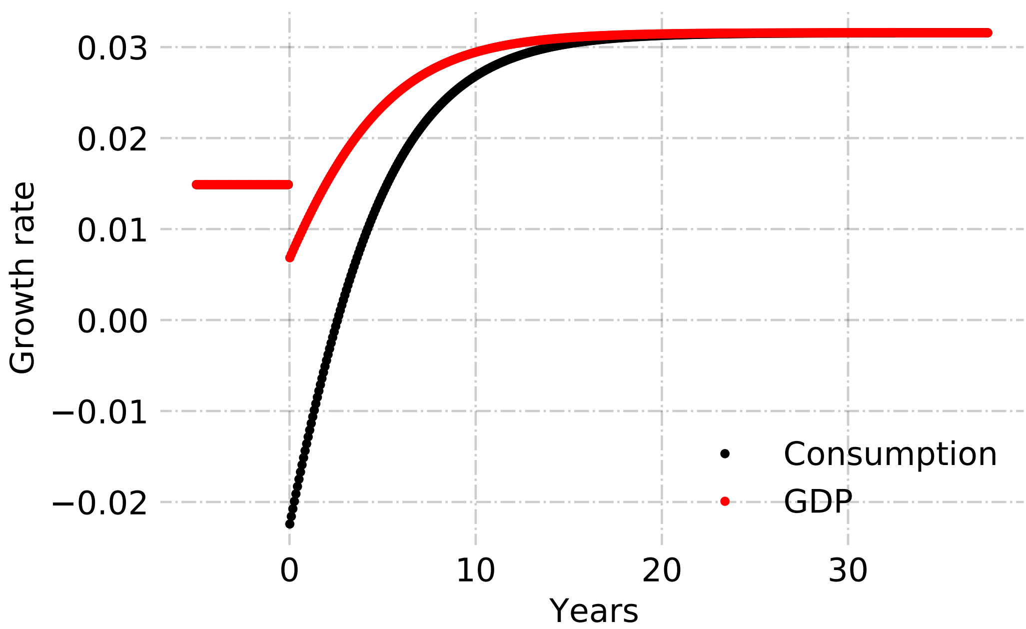
The change in steady state capital stock level is also in line with the analysis comparing the competitive equilibrium with the social planner. When restricting the investment sector innovation to CE levels, the steady state capital stock increases. Holding the innovation rate fixed in this sector maintains the user cost of capital at the market economy level. However, the social planner still corrects the monopoly pricing of the investment good. As a result, steady state capital stock increases relative to the competitive equilibrium. However, when the consumption sector innovation is restricted, and the social planner is free to the choose investment sector innovation, capital stock is lower than the competitive equilibrium level but higher than the social planner level. While the first finding is expected, the second appears to contradict my previous arguments. In fact, it does not. The investment sector innovation rate in the constrained social planner's solution is higher than the unconstrained social planner, and hence the user cost of capital is higher. We would expect a lower accumulation of capital, but we observe a higher accumulation of capital because a reduction in the consumption sector innovation frees up some labor which can be allocated to investment good production. This increased labor in the sector increases the marginal product of capital, resulting in an increase in capital accumulation relative to the unconstrained social planner problem.

To better understand the welfare gains of moving to the social optimum from the market economy, I solve for the transition path of the economy, of which the initial point is the balanced growth path of the market economy. Figure 2 depicts the growth rates of GDP and consumption in the social planner equilibrium over time.[19] The social planner allocates more labor to research and consumption decreases immediately. As the technological progress rate increases, so does the consumption growth rate. However, it takes years for the economy to have a higher consumption growth rate than the market economy balanced growth path. After 7 years, the consumption growth rate surpasses 1.5 percent and eventually reaches the long-run rate of 3.2 percent. The GDP growth rate also decreases initially, although not by as much as

[19]Section 6 explains the solution method in detail. In order to calculate GDP in the social planner economy with two sectors, I assume the relative price of the two sectors follows the market economy pricing.

the consumption growth rate, and then increases gradually to its long-run value of 3.2 percent. Initially, the increase in the growth rate of investment (expenditures) makes the GDP growth rate higher than the consumption growth rate. There are two countering forces that affect the investment growth rate. Discounted capital stock goes down under the social planner, which leads to a reduction in investment. However, because of an increase in the growth rate of the quality of investment goods, the investment growth rate goes up. The second force dominates and we observe an increase in the growth rate of investment and hence GDP falls by less than consumption. Eventually, as the consumption growth rate continues to increase and the investment growth rate continues to decrease, the GDP growth rate converges to 3.2 percent. The transition from the market economy balanced growth path to the social planner equilibrium generates approximately 21.5 percent in welfare gains, measured in consumption equivalent terms.

Figure 2: GDP and Consumption Growth Rates, Social Planner



Notes: Consumption and GDP growth rates over time. Until year 0, the economy grows at the market economy equilibrium rates. At time 0, the social planner takes control and the economy eventually converges to the balanced growth path of the social planner.

To understand the relative importance of the distortions described in Section 3.4, I add the distortions to the social planner allocation one by one, and report the results in Table 6. In line with the theoretical analysis of Section 3.4, monopoly power distorts innovation rates toward consumption sector. As predicted by my theoretical analysis, the inter-temporal spillover effect and the net profit consideration distort innovation rates towards the industry with the lower innovation rate. Introducing these distortions one by one into the social planner problem distorts the allocation of innovative resources toward the consumption sector because the innovation rate in the calibrated social planner problem is higher in the investment sector.

Table 6: Relative Industry Innovation Rates

	Consumption / Investment, τ_c/τ_x
Social planner	0.86
Social planner + monopoly power	0.98
Social planner + inter-temporal spillover	0.92
Social planner + net profit	0.95

Notes: Social planner industry innovation rate ratios when externalities introduced individually to the social planner problem.

6 Innovation Subsidies

As established in the previous section, the model features under-investment in innovation in both sectors. Building on this result, I analyze the role of R&D subsidies in increasing these innovation rates to their socially optimal levels. I show that long-run welfare can be increased substantially by providing R&D subsidies to incumbent and entering firms in each sector. Considering only time-invariant subsidies, welfare can be increased by as much as 20.3 percent over the long-run.

Previous sections have explained various distortions in the economy. This section focuses on the government's use of sector-specific innovation subsidies (financed through lump-sum taxation of households) to increase welfare, specifically, the rates at which innovative activities in each sector should be subsidized. I address these questions by finding the welfare maximizing *subsidy system*, where incumbents in the consumption sector are subsidized at a rate s_c, incumbents in the investment sector are subsidized at a rate s_x, entrants in the consumption sector are subsidized at a rate $1-(1-s_c)\gamma$, and entrants in the investment sector are subsidized at a rate $1-(1-s_x)\gamma$. Recall that, according to equations (17) and (18), innovation in, say, the consumption sector will be allocated efficiently between incumbents and entrants if incumbents are subsidized at a rate s_c and entrants at a rate $1-(1-s_c)\gamma$. The analysis is restricted to entry and incumbent firm R&D subsidies that result in the optimal within-sector allocation of innovative resources across entering and incumbent firms, so that welfare comparisons of the different subsidy systems reflect welfare changes due to total innovation in each sector.

Starting from the balanced growth path of the benchmark economy (described in the calibration section), I alter the subsidy rates (unanticipated by agents in the economy) for all the subsequent times and keep them constant. I then calculate the transition to the new balanced growth path under the new subsidy system. Finally, I calculate the welfare gain/loss of the subsidized economy relative to the benchmark economy. The algorithm behind the subsidy-system welfare impact calculations is described below:

1. Discount the variables that grow at the balanced growth path with the technology indices that leads to growth.
2. Solve for the steady states of the benchmark economy and the subsidized economy.
3. Using the reverse shooting algorithm described by Judd (1998), solve the transition of the economy from the steady state of the benchmark economy to the steady state of the subsidized economy.
4. Starting from the steady state of the benchmark economy—and normalizing the technology indices at this steady state equal to one—simulate the economy forward and generate the (non-discounted) consumption sequence. Retain the two consumption sequences that will be used to compute welfare gain: (1) the consumption sequence of the benchmark economy; and (2) the consumption sequence of the subsidized economy.
5. Calculate the sum of discounted utility of these two consumption sequences. The sum of the discounted utility of the benchmark economy at the balanced growth path is given in equation (25), with C_0 denoting the level of consumption at the time of subsidy change and g_C denoting the consumption growth rate:

$$W(C_0, g_C) = \frac{1}{\rho}\left(\ln C_0 + \frac{g_C}{\rho}\right). \tag{25}$$

   The sum of discounted utility of the subsidized system is calculated using numerical integration over the utility values of the consumption sequence.
6. Calculate the consumption equivalent welfare change described in Equation (26):

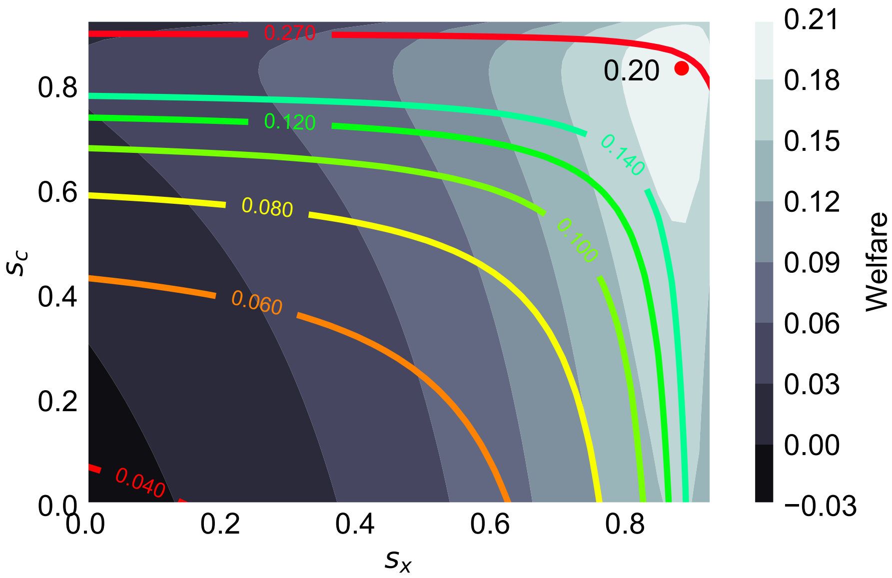
$$W(\xi C_0, g_C) = \int_0^\infty \exp(-\rho t)\ln(C_t^s)dt, \tag{26}$$

   where C_t^s is consumption at time t in the subsidized economy. The welfare gain/loss is equal to $\xi - 1$, the rate of increase in consumption in the benchmark economy that makes the representative household indifferent between the benchmark and subsidized economies.

Figure 3 depicts the welfare gains from a wide range subsidies as a contour map. Contour shades represents welfare gains from subsidizing R&D expenses of consumption sector incumbents at a rate s_c (y-axis), R&D expenses of investment sector incumbents at a rate s_x (x-axis), consumption sector entrants at a rate $1-(1-s_c)\gamma$, and investment sector entrants at a rate $1-(1-s_x)\gamma$. The curves on top of contour shades show total R&D labor in the economy at the balanced growth path. The total amount of R&D expenditures that these subsidies induce is shown in Appendix C.

There are several results worth highlighting. First, holding the subsidy of a sector constant as the subsidy of the other sector increases, the welfare gain increases until

Figure 3: Welfare Gain



Notes: Contour map of welfare gains of R&D subsidies. The curves on top of contour shades show total R&D labor in the economy at the balanced growth path. The red point shows the welfare maximizing subsidy rates to incumbent firms, with 0.20 indicating the corresponding welfare gain.

a critical point beyond which further subsidies reduce the welfare gain. The same result holds when subsidies to both sectors increase simultaneously. Recall, from the distortions highlighted in Section 3.2, that the competitive equilibrium innovation rate can be above or below the social planner innovation rate. In this particular economy, the competitive equilibrium innovation rate falls below the socially optimal innovation rate; by definition, raising innovation rate to socially optimal levels leads to higher welfare. When the innovation rates surpass the optimal levels, welfare gains start decreasing. Second, the maximum welfare gain is attained by subsidizing the consumption sector R&D at 83.5 percent and investment sector at 88.3 percent, which results in about 20.3 percent welfare gain and 3.2 growth rate of the economy in the long run, suggesting that the level of under-investment in innovation is quite high in both sectors. Third, the iso-welfare curves are tilted towards investment sector R&D subsidy. A given rate of subsidy generates higher welfare gain when it is applied to only the investment sector than when it is applied only to the consumption sector.

Correcting the distortions requires more than 80 percent in R&D subsidies to each sector for two main reasons. First, as explained in the Section 5, there are large distortions in the market economy, resulting in under-investment in innovation. Such under-investment is typical in models based on Klette and Kortum (2004).[20] Second, subsidizing R&D intensifies the inter-temporal spillover effect, which tends to reduce

[20]Lentz and Mortensen (2015) show that the social planner increases innovative resources threefold. Similarly, Segerstrom (2007) find that innovation should be heavily subsidized.

market economy innovation rates relative to the social optimum. Subsidizing R&D also promotes a higher entry rate by increasing the value of firms. The higher entry rate corresponds to a higher probability for an incumbent firm to shrink by one good, intensifying the inter-temporal spillover effect and thus discouraging innovation by incumbent firms. Firm R&D therefore needs to be subsidized further to counteract this inter-temporal spillover.

In Appendix E, I conduct further analyses on the role of entry in necessitating higher subsidy rates. In Appendix E.1, I calibrate my model using its implications for job destruction rates rather than job creation rates as in Section 4. This alternative calibration generates slightly lower entry rates for both industries. The optimal R&D subsidy rates in both industries under this alternative calibration are also slightly lower than those in the benchmark calibration in Section 4.

In Appendix E.2, I conduct comparative statics on the entry cost parameters, ψ_c and ψ_x, which alter the entry rates. As the share of entrant innovation in total innovation falls, so do the socially optimal innovation rates and the welfare gain of moving to socially optimal allocation.

In Appendix E.3, I subsidize only the incumbent firms while keeping entry subsidy/tax rates at the benchmark calibration levels in Section 4. This incumbent-only R&D subsidy exercise increases incumbent innovation rates while reducing the entry rates and hence the inter-temporal spillover effect. The optimal subsidy rates to incumbents in both industries are approximately 8 percentage points lower in the incumbent-only R&D subsidy exercise than in the exercise in which both incumbents and entrants are subsidized. Therefore, the intensification of the inter-temporal subsidy effect in the optimal subsidy analysis partly explains the large optimal subsidy rates. However, R&D subsidy rates remain large in the incumbent-only exercise, suggesting that other distortions are also important in necessitating large optimal subsidy rates.

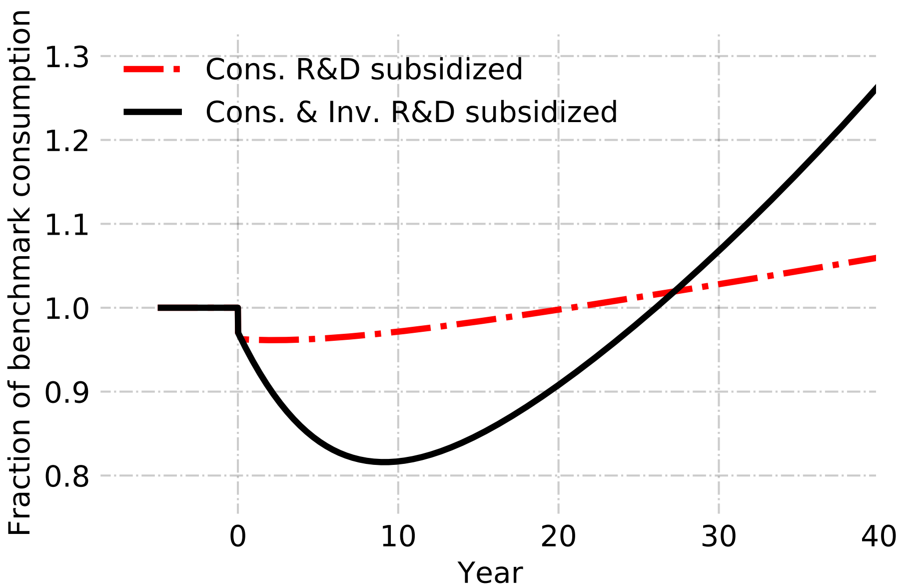
How does this economy achieve the maximum welfare gain? Analyzing the trajectory of consumption helps us to answer this question. Figure 4 shows the trajectories of consumption in two scenarios: (i) the *Consumption Sector Subsidized Economy*, which applies an 84 percent subsidy to consumption sector R&D by incumbents and a 59 percent subsidy to the cost of entry in the consumption sector, and (ii) the *Optimally Subsidized Economy*, which applies the socially-optimal subsidy system (84 percent to consumption sector R&D, 88 percent to investment sector R&D, 59 percent to entry into the consumption sector, and 71 percent to entry into the investment sector.)[21]

For a better comparison of consumption paths after the subsidy to the economy, I discount each consumption path in the figure with the benchmark economy consumption (the balanced growth path of the economy described in Section 4). Allocating more research labor to innovation results in a reduction in the production of

[21] Recall that entrant innovation is subsidized at a lower rate to correct for the congestion externality during entry.

consumption goods in earlier periods but a higher long-run consumption growth rate. Consumption in the Consumption Sector Subsidized Economy rebounds more quickly. However, consumption in the Optimally Subsidized Economy surpasses the Consumption Sector Subsidized Economy in later years. Consumption grows more slowly in earlier periods in the Optimally Subsidized Economy because of the response of capital to subsidies. Specifically, subsidizing investment sector R&D leads to higher innovation rates in this sector. This leads to a lower growth rate of the price of investment goods (higher in absolute terms) and a higher user cost of capital. Therefore, capital accumulates slowly. Hence, in earlier years, consumption grows at a lower rate when the investment sector is subsidized. Later on, after the economy reaches the balanced growth path, the higher innovative step of investment sector generates a higher consumption growth rate. Therefore, consumption in this economy catches and surpasses the benchmark economy and Consumption Sector Subsidized Economy.

Figure 4: Sequence of Consumption with Different Subsidies

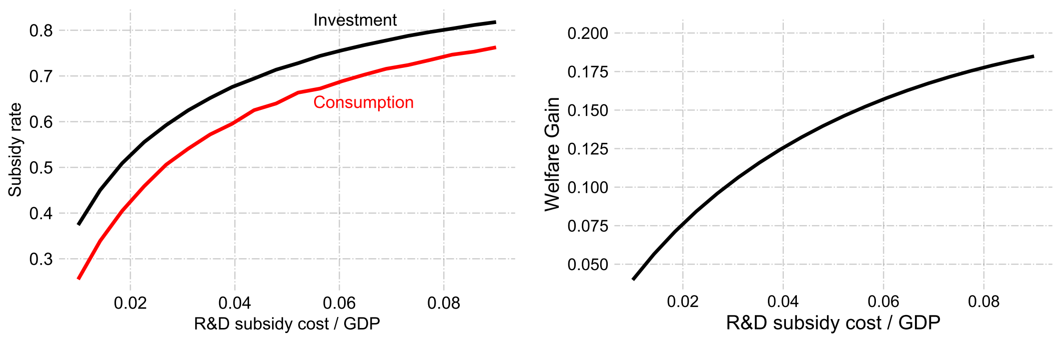


Notes: Consumption paths are relative to benchmark economy balanced growth path consumption levels. *Cons R&D Subsidized*: Consumption R&D subsidy of 84% with 59% subsidy to entrants. *Cons & Inv R&D subsidized*: The subsidy system that maximizes welfare gains (84 percent to the consumption sector incumbents, 88 percent to investment sector incumbents, 59 percent to consumption sector entrants, and 71 percent to investment sector entrants.

6.1 Welfare Gains with Limited Transfer Budget

Subsidizing incumbent R&D to maximize welfare gains requires taxes on the order of 16 percent of GDP. The model abstracts from two issues that render this amount unreasonable: the distortionary effects of taxation, and the political economy of taxation. With this in mind, it is worth exploring the fiscal authority's optimal allocation of subsidies across sectors when its transfer budget is constrained by some exogenous factors. On the one hand, a given investment sector subsidy rate leads to higher welfare gains than an equivalent subsidy to the consumption sector. On the other hand, any given investment sector subsidy costs more than the equivalent consumption sector subsidy. In this economy, the welfare gain advantage of investment sector dominates. For example, if the tax authority increases its *incumbent R&D subsidy budget* – defined as the GDP share of the cost of incumbent firm R&D subsidies – by 10 percent (from 0.2 percent of GDP to 0.23 percent of GDP), it can achieve a 0.2 percent welfare gain by taxing the consumption sector R&D by 1.1 percent and subsidizing the investment sector R&D by 15.3 percent.

Figure 5: Cost Constrained Optimal R&D Subsidy



Notes: Optimal R&D subsidies to sectors under limited transfer budget and associated welfare gain.

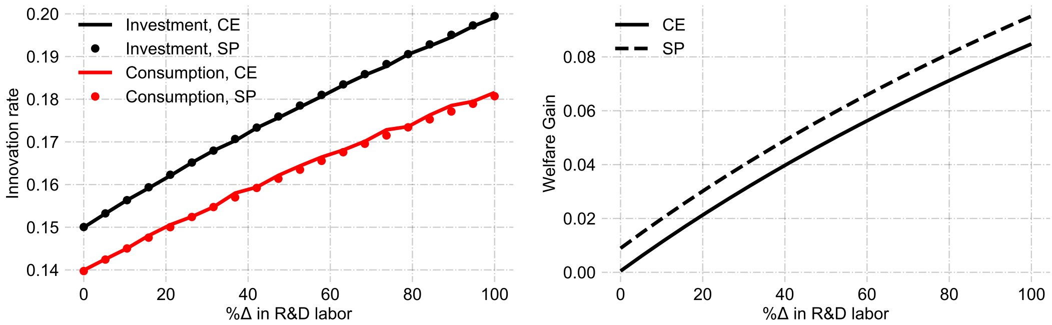
The left panel of Figure 5 shows the optimal government R&D subsidy rate to the two sectors as a function of incumbent R&D subsidy budget, defined as the GDP share of the cost of incumbent firm R&D subsidies, at the balanced growth path. It is always optimal to subsidize the investment sector at a higher rate than the consumption sector. The right panel of Figure 5 shows the welfare gains associated with optimal R&D subsidies under a limited transfer budget. As the incumbent R&D subsidy budget increases, the welfare gains in consumption equivalent terms increases, but at a decreasing rate.

A similar exercise, depicted in Figure 6, analyzes the optimal government subsidy and social planner allocations when the total amount of labor allocated to R&D is constrained. For instance, with R&D labor constrained to the total R&D labor in the BGP of the competitive equilibrium, the social planner can still generate a 0.9 percent welfare gain by increasing the innovation rate in the investment sector from 14.6 percent to 15 percent and reducing the innovation rate in the consumption sector from 14.9 percent to 14 percent.[22] As expected, relaxing the total R&D labor constraint yields additional welfare gains. For example, the constrained social planner allocation that increases the total R&D labor by 10 percent leads to a 2 percent welfare gain.

[22]The social planner also corrects the distortion in the Euler equation as a result of investment goods producers' monopoly power, and this correction contributes to the welfare gain.

In this new allocation, the innovation rate in the investment sector is equal to 15.6 percent and that in the consumption sector is equal to 14.5 percent.

Figure 6: Labor Constrained Optimal R&D Subsidy



Notes: Constrained optimum sectoral innovation rates under an R&D labor constraint, and the associated welfare gains. %Δ R&D labor represents the increase in the total R&D labor relative to the BGP of the benchmark economy, in percentages. CE stands for competitive equilibrium and SP stands for social planner.

The left panel of Figure 6 depicts innovation rates resulting from the constrained optimum government subsidy and social planner allocations, whereas the right panel shows the welfare gains of such government policies and social planner allocations. Both the constrained optimum government subsidy and the social planner achieve the same innovation rates. Furthermore, in both the social planner allocation and the competitive equilibrium, the innovation rate in the investment sector is higher than the innovation rate in the consumption sector. The social planner achieves higher welfare gains than the government subsidy. This partially stems from the social planner's ability to correct the distortions in the Euler equation, set time varying innovation rates on the transition, and the social planner's greater flexibility when dealing with multiple distortions in the innovation process.[23]

In this section I argue that the government should subsidize investment sector innovation at a higher rate than the consumption sector. Similarly, the social planner sets a higher innovation rate in the investment sector. However, this result may seem contrary to the results of Section 3.4, in which I argue that the social planner sets a higher innovation rate in the consumption sector. In the theoretical analysis of Section 3.4, I assume the industries have identical innovation functions, and capital is not used in investment good production. When industries have identical innovation functions, and industries differ mainly with respect to their location in the supply chain, it is optimal to set higher innovation rates in the consumption sector as it is closer to the final consumption and hence has higher *influence*. However, in this section, the investment sector has a larger innovative step, that is, $\lambda_x > \lambda_c$. A higher quality improvement in the investment sector following successful innovation makes it socially optimal to subsidize that sector at a higher rate.

[23]Recall that I consider only constant subsidy rates over time.

7 Comparison to Atkeson and Burstein (2019)

A key contribution of my model is incorporating an investment good sector, distinct from the consumption sector, into Klette and Kortum (2004) (KK) model. To highlight the importance of this heterogeneity in optimal R&D policy design, I compare my results with those obtained using the Atkeson and Burstein (2019) (AB) methodology.

The AB model nests many endogenous and semi-endogenous growth models including the KK model. It has a rich structure in many aspects. For example, it incorporates both expanding variety and quality ladder type innovations as well as own good innovations among incumbent firms. AB develop a methodology to linearly approximate output and productivity trajectories. Using this methodology, AB analyze changes in the output and productivity trajectories following a policy-induced change in the economy's innovation intensity, and how these responses vary based on the degree of inter-temporal knowledge spillovers.

My model and analyses differ from AB in two key ways. First, despite its broad scope, the AB model does not nest my model. Specifically, the AB model assumes that the allocation of innovative resources is conditionally efficient; as described in Section 3, externalities in the innovation process and differences in markups and innovation functions across sectors lead to the misallocation of innovative resources across sectors. In my model, the social planner can therefore increase welfare by reallocating a given level of innovative resources across sectors. In particular, the constrained optimum social planner allocation increases welfare by 0.9 percent by increasing the innovation rate in the investment sector and reducing the innovation rate in the consumption sector, while keeping the total labor allocated to R&D at the competitive equilibrium level. On a related note, I analyze sector-dependent optimal government R&D subsidy policy, which lies outside the scope of AB. AB does not analyze the impact of non-proportional changes in innovation subsidies when the allocation of innovative resources across firms at the initial BGP is conditionally inefficient.

To compare my results with AB methodology, I analyze the impact of a policy-induced 10% increase in R&D labor on the economy in four cases: (i) a one-sector version of my model where consumption and investment goods are produced by the same sector and the model is solved using the AB approximation; (ii) a one-sector version of my model where consumption and investment goods are produced by the same sector and the model is solved non-linearly; (iii) a two-sector version of the model with sector-dependent R&D subsidies; and (iv) a two-sector version of the model with a uniform subsidy across sectors. Since the one-sector version of my model is nested in the AB model, the AB methodology is well-suited to solving for the transition path of the economy. Finally, where applicable, I appeal to AB's results on endogenous growth models.[24]

[24]Specifically, I rely on Proposition 2 of Atkeson and Burstein (2019). To a first-order approximation, the new path of productivity, $\{Z'_t\}_{t=1}^{\infty}$, is equal to $\log Z'_{t+1} - \log \bar{Z}_{t+1} \approx \sum_{j=0}^{t} \Theta_e(\log l'_{rt-j} - \log \bar{l}_r)$, where Θ_e is the impact elasticity of the economy with respect to entrant innovation, and l_r is the R&D labor (variables with bar represent initial BGP values). In my model, $\Theta_e = \frac{z^{1-\gamma}}{\gamma\psi\chi} \ln \lambda(\psi\chi z^{\gamma} + \chi b^{\gamma})$.

In each case, the chosen subsidy rates maximize consumption-equivalent welfare gains by increasing R&D labor in the new BGP less than 10% of the initial BGP. Table 7 lists the subsidy rates, the resulting BGP growth rates, and the consumption-equivalent welfare gains in each case. Figure 7 plots log difference of sectoral productivity relative to the initial BGP productivity, and the log difference of output relative to the initial BGP output in each case.

Table 7: Policy Exercises

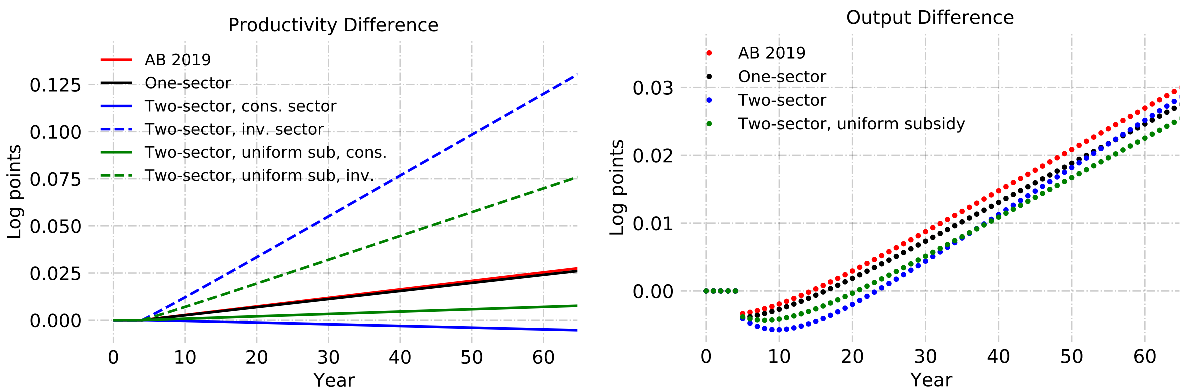
Case	R&D Subsidy	BGP Growth Rate	Welfare Gain
i) One-sector (AB methodology)	17.9% uniform	–	–
ii) One-sector (non-linear)	17.9% uniform	1.55%	1.28%
iii) Two-sector	Consumption: 4.9% Investment: 22.2%	1.56%	1.12%
iv) Two-sector	Consumption: 17.5% Investment: 17.5%	1.55%	1.07%

Notes: The impact of a policy-induced 10% increase in R&D labor on the economy in four cases: (i) a one-sector version of my model where consumption and investment goods are produced by the same sector and the model is solved using the AB approximation; (ii) a one-sector version of my model where consumption and investment goods are produced by the same sector and the model is solved non-linearly; (iii) a two-sector version of the model with sector-dependent R&D subsidies; and (iv) a two-sector version of the model with a uniform subsidy across sectors. In each case, the chosen subsidy rates maximize consumption-equivalent welfare gains by increasing R&D labor in the new BGP less than 10% of the initial BGP.

As seen in the left panel of Figure 7, the linear approximation of productivity using the AB methodology closely tracks the non-linear solution of the productivity trajectory in the one-sector version of my model. A permanent increase in the R&D subsidy from 10% to 17.9% leads to a reduction in output relative to the initial BGP in earlier periods, and an increase in output in later periods. Output using the AB approximation remains within 0.3% of the non-linear solution in each of the 60 years after the policy change. While hardly surprising since this case is nested by Atkeson and Burstein (2019), it is nonetheless reassuring to establish the closeness of the two solutions.

In general, optimal subsidies differ from those implied by the AB methodology whenever the allocation of innovative resources across sectors is conditionally inefficient. Case (iii) corresponds to such an environment, where a 22.2 percent R&D subsidy to investment sector incumbents, and a 4.9 percent R&D subsidy to consumption sector incumbents result in a 10 percent increase in R&D labor. In turn, this leads to an approximately 13% increase in investment sector productivity over 60 years, and a reduction in consumption sector productivity relative to the initial BGP.

Figure 7: Productivity and Output Dynamics



Notes: Trajectories of productivity and output after a policy induced 10% increase in R&D labor.

Immediately after the policy change, output falls short of initial BGP output as a result of lower production labor and lower investment. Eventually, increased productivity in the investment sector drives output above the initial BGP levels, the economy converges to a 1.56% growth rate, and experiences a 1.12% consumption-equivalent welfare gain.

Comparing cases (iii) and (i) shows that the trajectories of the two economies differ mostly in years 10-30. One year after the change in innovation subsidy policy, output in case (iii) is 0.07% lower than output in the AB approximation. Fifteen years after the policy change, predictions of output with the AB approximation are about 0.49% higher than those in the model with sectoral heterogeneity. Long-run trajectories are closer, with the BGP output growth rate at 1.56% in case (iii) and 1.55% in case (ii).

In case (iv), I consider a uniform subsidy – 17.5% R&D subsidy to incumbent R&D in each sector – that increases R&D labor by 10%. Trajectories of sectoral productivity differ from that of case (iii). Output trajectories also differ, with higher BGP growth rate in the sector-dependent subsidy. In the calibration of the model, λ_c is low, and subsidizing consumption sector at a higher rate does not lead to substantial changes in the economy.

Overall, I confirm that the AB methodology provides a powerful tool to approximate output and productivity trajectories following a policy-induced changed in the amount of research labor in a one-sector economy when innovative resource allocations are conditionally efficient. However, these techniques do not apply to the baseline model, which features heterogeneous sectors and a conditionally inefficient allocation of innovative resources across sectors.

8 Conclusion

I analyze the heterogeneity of innovative activity across sectors in a quantitative environment where firm–level innovation is the main driver of the long–run macroeconomic growth. I consider two policy questions in this environment. First, how should a government target the different sectors through R&D subsidies to increase welfare? Second, how should the government allocate a limited transfer budget?

To answer this and related questions, I develop a quality ladder type of model based on the framework of Klette and Kortum (2004) that features two sectors: consumption goods producers and investment goods producers. These sectors differ mainly in their output's use, R&D cost functions, and quality ladder steps sizes. I calibrate my model using its firm dynamics implications and US data on job creation and destruction. In the quantitative exercise, I construct a consumption goods and investment goods industry based on the Input-Output linkages of industries. An interesting result of the calibration exercise is that investment sector firms are more innovative, have a higher quality ladder step, but have a higher cost of innovation.

A sector's contribution to macroeconomic growth and welfare of the society depends on the sector's position in the supply chain, its innovation rate, and the quality increase (or cost reduction) of the goods after a successful innovation in the sector. Consumption sector innovation affects consumption growth directly, whereas investment sector innovation affects consumption growth indirectly through its effect on the capital stock of the economy. In the calibrated model, the consumption sector generates about the same amount of innovation as the investment sector. In this sense, the consumption sector contributes more to growth. However, the investment sector is more innovative. Once it innovates, it increases the quality of existing goods more than the consumption sector. The number of innovations on, say, central processing units (CPUs), are lower than the number of innovations on, say, restaurants. However, once a better CPU is developed, its quality increase is higher than the quality increase of better restaurant food.

The Schumpeterian innovation process described in the model leads to various distortions in the economy, resulting in innovation rates in both sectors that are lower than socially desirable levels and leaving room for government intervention. I consider two possible interventions. First, the government can increase welfare in the long run by subsidizing R&D in each of the sectors. I show that a given rate of R&D subsidy to the investment sector generates more welfare gain than an equal amount of R&D subsidy to the consumption sector. I also show that the welfare gain from the optimal subsidy system can reach up to 20 percent in consumption equivalent terms. Second, I analyze constrained optimum subsidy system if the government can allocate a limited transfer budget across different sectors. I show that a subsidy system tilted toward the investment sector generates more welfare gain than a uniform subsidy system with the same overall cost.

There are some caveats to my findings. First, subsidizing R&D at a rate as high as 88 percent might be politically infeasible. For reasons exogenous to my model, taxpayers may not desire such high levels of transfers to business owners. R&D subsidies to incumbent firms constitute about 16 percent of GDP. A tax authority may not have the political power to collect so much from taxpayers and distribute it to businesses. Even though the optimal R&D subsidy rates suggested by my model might not be politically feasible, they are informative about the extent of externalities in the consumption and investment goods industries. Moreover, I address these concerns by conducting constrained–optimum subsidy analysis and show that there are economically meaningful gains from R&D subsidies by increasing the R&D subsidy budget by more politically feasible amounts and by subsidizing R&D in the investment sector at a higher rate than R&D in the consumption sector.

Second, I also abstract away from moral hazard. Firms may label their operating expenses as R&D in order to benefit from R&D subsidies. For example, Chen et al. (2018) show that some firms relabel their expenses as R&D in China to benefit from R&D subsidies. However, in their literature review on the effectiveness of government policies in promoting innovation, Bloom et al. (2019) argue that R&D subsidies actually promote outputs of innovation (such as patents), although they do not rule out the mislabeling of expenses as R&D. Therefore, even though moral hazard in the form of mislabeling of expenses as R&D cannot be ruled out, an optimal subsidy analysis ignoring the moral hazard problem does provide insights in designing optimal R&D subsidies.

Third, many of the results rely on the estimates of the quality ladder steps, which are identified by four statistics: the consumption growth rate, the growth rate of the relative price of investment goods, and the innovation rates in each industry. Any systematic mismeasurement of these statistics would bias the results. For example, the results would not be accurate if the growth rate of the relative price of investment goods was affected by factors other than the quality increase. Similarly, in the model, the only source of quality improvement is dedicated R&D activity. In this sense, innovation in my model is regarded in the broadest sense: any activity that leads to a quality improvement is considered as innovation. Nonetheless, my results seems comparable with other studies in the literature.

Overall, despite these caveats, the analyses in the paper provide insights about the magnitudes of externalities in the innovation process, and the design of industry-specific R&D subsidy system with and without constraints on the amount of transfers a tax authority can make to R&D performing firms.

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A Calibration of Sector Dependent Parameters

I target the following sector–level variables in my calibration: the innovation rates among entrants, and the innovation rates among incumbents. In contrast to the stylized model, it is impossible to neatly classify sectors in the data as consumption or investment; output from a given sector serves both roles. Moreover, output of a sector can be used as intermediate inputs, which is entirely missing in the stylized model. I therefore apply the following procedure to construct sector-level targets in my model:

1. Using Business Dynamics Statistics (BDS), compute innovation rates by entrants and incumbents in each sector classified according to SIC system.
2. If a sector level data is not in I-O classification system, using the crosswalk described below, convert the industry level data with SIC classification system into industry level data with I-O classification system.
3. Using the weights constructed from Input-Output tables, compute final good industry targets as the weighted average of I-O industry values.

A.1 Industry Level Targets

In my model, since each product line in a sector employs the same amount of labor, there is a one to one relation between job creation rate and innovation rate in an industry, i.e., the ratio of the number of jobs created by entering firms to total employment in a sector is equal to the innovation rate among entrants. Similarly, the ratio of the number of jobs created by expanding firms to total employment in a sector is equal to the innovation rate among incumbents in that industry. Therefore, at the industry level, I target job creation rates by entering establishments, and job creation rates by expanding establishments obtained from Business Dynamics Statistics (BDS) under the identifying assumption that an establishment in the data corresponds to a firm in my model.

A.2 Crosswalk into I-O Industry Classification

While BDS follows the SIC industry classification, the I-O data are classified according to I-O system, a variant of NAICS. I use the following crosswalk to convert SIC industry data to I-O industry data. Let $M_{i,t,SIC}$ be the value of any variable, M, in SIC industry i in year t. I construct the corresponding value in NAICS super-sector s, denoted $M_{s,t,NAICS-SUPER}$, as the employment–weighted share of M across SIC industries:

$$M_{s,t,NAICS-SUPER} \equiv \sum_i e_{s,i} \, M_{i,t,SIC},$$

where $e_{s,i}$ is the percentage of employment in NAICS industry s coming from SIC industry i in the first quarter of 2001 (Bureau of Labor Statistic (BLS)[25]). Some NAICS super-sectors are in one-to-one correspondence with 2-digit I-O industries, while in other cases, a NAICS super-sector corresponds to a combination of I-O industries. I disaggregate the latter group into 2-digit I-O industries, $M_{j,2-digit}$, using Current Employment Statistics data on employment shares of 2-digit industries in NAICS super-sectors in first quarter of 2001 from BLS:

$$M_{j,t,2-digit} = e_{j,s} M_{s,t,NAICS-SUPER},$$

where $e_{j,s}$ is the employment share of 2-digit I-O industry j in NAICS super-sector s in the first quarter of 2001.

A.3 Construction of ω_i^c and ω_i^x

To construct the targets for final goods producing industries in my model, I take a weighted average of industry level values, where the weights are the industry-labor requirements to produce one unit of final good. Let M_c and M_x be calibration targets for final consumption and final investment industry. Then, $M_c = \sum_j \omega_j^c M_j$, and $M_x = \sum_j \omega_j^x M_j$, where M_j is the value of the particular target M observed in data for I-O industry j.[26] Let ω_j^i denote the labor from sector j required to produce one unit of final good in sector $i = c, x$ as a fraction of the total unit labor requirement in sector i.

The parameters ω_j^c and ω_j^x are constructed as follows:

1. Calculate the industry-labor requirements to produce one unit of final output in each industry, L^c and L^x:

$$L^c = l(I - B)^{-1}C, \quad L^x = l(I - B)^{-1}X,$$

where l is a diagonal matrix with elements $l_{jj} = \frac{N_j}{Y_j}$, the ratio of employment in industry j, N_j, to gross output in industry j, Y_j; I is the identity matrix; B is input-output matrix with elements $B_{ij} = \frac{y_{ij}}{Y_j}$ representing the ratio of intermediate input use of industry j from industry i, y_{ij}, to the gross output of industry j, Y_j; C is a vector of consumption shares across industries,

$$C_j = \frac{\text{Household consumption of output of industry } j}{\sum_j \text{Household consumption of output of industry } j};$$

[25]See: https://www.bls.gov/ces/cesnaics02.htm

[26]These industries are classified according to the I-O system, a variant of NAICS, and correspond to mining and logging, utilities, construction, manufacturing, wholesale trade, retail trade, transportation and warehousing, information, education and health services, leisure and hospitality, other services, financial activities, professional and business services.

and X be a vector of investment shares across industries,

$$X_j = \frac{\text{Output of industry } j \text{ used as investment}}{\sum_j \text{Output of industry } i \text{ used as investment}}.$$

2. Using industry-labor requirements, I calculate final good weight of an industry as $\omega^i_j = L^i_j / \sum_j L^i_j$, for $i = c, x$. The data used to construct the above stated variables are obtained from Bureau of Economic Analysis Input-Output use tables after redefinitions.

I then employ the algorithm described above to find sector level aggregates such as the number of jobs created by entrants and incumbents, employment, compensation of employees, value added, etc. Note that the financial services industry (FIRE) is included in the I-O table to construct the labor content of the final goods, but is dropped from the analysis while constructing weights, and hence targets for the final goods.

B Details of the Model

B.1 Consumption Goods Producers

I define the problem of a differentiated consumption good producer in two steps. First, I define the static problem: how much to produce, and its demand for factor inputs. After solving this problem and establishing the profit from production, I turn to the dynamic problem: how much to invest in R&D to maximize the value of the firm.

Each production unit of a firm has a Cobb-Douglas production function with capital elasticity α. Production of each unit is independent of any other production units a firm may possess. Hence, each production unit solves the following cost minimization problem:

$$\min_{l_c, k_c} wl_c + rk_c \quad \text{subject to} \quad k_c^\alpha l_c^{1-\alpha} = c,$$

where w and r are the market wage and capital rental rates. The resulting cost function, $C^p((w,r),c) = \frac{r^\alpha w^{1-\alpha} c}{\tilde{\alpha}}$, with $\tilde{\alpha} \equiv \alpha^\alpha (1-\alpha)^{1-\alpha}$, is common across all the production units in a sector. As a result, Bertrand competition yields a price $p^j = \lambda_c \frac{r^\alpha w^{1-\alpha}}{\tilde{\alpha}}$ for differentiated product j. Using this price and the demand function for differentiated goods yields the profit of a differentiated good producer:

$$\pi = pc - C^p((w,r),c) = \left(1 - \frac{1}{\lambda_c}\right) Z, \tag{27}$$

where Z equals aggregate consumption expenditure. Note that profits do not vary across differentiated goods in a sector.

Turning to the dynamic problem, I use the profit function in (27) to derive the firm's value function, which can be expressed either as a function of the level of research labor or, more conveniently, the level of innovation arrival rate per good. Let $c(b_c) \cdot m$

denote the level of l_R implicitly defined by $\varphi(m, l_R) = \beta$, where $b_c \equiv \beta/m$. Since $\varphi(\cdot)$ is strictly increasing in l_R, and homogeneous of degree one, $c(b_c)$ is well-defined and convex in b_c. For concreteness, I assume $c(b_c) = \chi_c b_c^\gamma$, where $\chi_c > 0$ is a scale parameter.

The Bellman equation of the firm on the balanced growth path is

$$\begin{aligned} RV(m,Z) &= \max_{b_c \geq 0} \Bigg\{ \left(1 - \frac{1}{\lambda_c}\right) mZ - (1 - s_c^i) w \phi_c(b_c, m) + \frac{\partial V(m,Z)}{\partial Z} \dot{Z} \\ &+ m b_c [V(m+1, Z) - V(m,Z)] + m\tau_c [V(m-1,Z) - V(m,Z)] \Bigg\}, \end{aligned}$$

where s_c^i is the rate of R&D subsidy for the consumption sector incumbents, and τ_c is the equilibrium Poisson innovation arrival rate in the consumption sector. Given that firm profits, and R&D expenditures are linear in the number of goods, I conjecture that $V(m, Z) = \nu_c m Z$ for some $\nu_c > 0$ and verify this claim. Inserting the guess yields

$$R\nu_c m Z = m \left(1 - \frac{1}{\lambda_c}\right) Z - (1 - s_c^i) w m \chi_c b_c^\gamma + \nu_c m Z g_Z + m b_c \nu_c Z - m \tau_c \nu_c Z,$$

where b_c is the optimal innovation intensity, and $g_Z \equiv \frac{\dot{Z}}{Z}$ is the growth rate of household consumption expenditure.

B.2 Solution of the Model

The representative household maximization problem is described in Section 2.1. Consumption is a quality adjusted aggregate of differentiated consumption goods described in equation (1). Since I solve for a symmetric equilibrium and assume limit pricing, the highest quality versions of each differentiated consumption product gets the same positive demand, and the lower quality versions have a demand of zero. This demand function is described in equation (2). Then we simplify equation (1) into

$$C = \exp\left(\int_0^1 \ln\left(q(\omega)c(\omega)\right) d\omega \right), \tag{28}$$

where $q(\omega)$ is the highest quality level of product ω, and $c(\omega)$ is the consumption of product ω with highest quality. Also, using the fact that (i) the production function of differentiated goods in a sector is identical; (ii) products face symmetric demand; and (iii) the labor hired, l_c, and capital rented, k_c, does not vary across differentiated goods, the equilibrium consumption of each differentiated unit is $c(\omega) = k_c^\alpha l_c^{1-\alpha}$, where k_c and l_c do not depend on the product. Therefore, in equilibrium, aggregate consumption simplifies to

$$C = \exp\left(\int_0^1 \ln\left(q(\omega)k_c^\alpha l_c^{1-\alpha}\right)d\omega\right) \tag{29}$$
$$C = k_c^\alpha l_c^{1-\alpha}\exp\left(\int_0^1 \ln\left(q(\omega)\right)d\omega\right) \tag{30}$$
$$C = k_c^\alpha l_c^{1-\alpha}Q_c, \tag{31}$$

where $Q_c = \exp\left(\int_0^1 \ln\left(q(\omega)\right)d\omega\right)$ is the average quality in the consumption sector. Equation (31) will be used to determine the growth rate of consumption on the balanced growth path. The average quality-adjusted price of the consumption good is equal to

$$P_c = \exp\left(\int_0^1 \ln\frac{p(\omega)}{q(\omega)}d\omega\right) \tag{32}$$
$$= \exp\left(\int_0^1 \ln\frac{\frac{\lambda_c r^\alpha w^{1-\alpha}}{\tilde{\alpha}}}{q(\omega)}d\omega\right) \tag{33}$$
$$= \frac{\lambda_c r^\alpha w^{1-\alpha}}{\tilde{\alpha}}\frac{1}{Q_C}. \tag{34}$$

Again, this is a result of identical innovative steps and identical production functions. I normalize the price of the consumption good to 1:

$$1 \equiv P_c = \lambda_c \frac{r^\alpha w^{1-\alpha}}{\tilde{\alpha}Q_c} \tag{35}$$

Similarly, investment is a quality adjusted aggregate of differentiated investment goods. Using the same arguments as above, the demand function of differentiated investment goods can be substituted into the investment aggregator and combined with the identical production functions of differentiated investment goods, so that aggregate investment simplifies to

$$X = k_x^\alpha l_x^{1-\alpha}Q_x, \tag{36}$$

where $Q_x = \exp\left(\int_0^1 \ln\left(q(\omega)\right)d\omega\right)$ is the average quality in the investment sector. The quality-adjusted average price of the investment good is also equal price of each differentiated good:

$$P_x = \lambda_x \frac{r^\alpha w^{1-\alpha}}{\tilde{\alpha}Q_x}. \tag{37}$$

The two remaining first order conditions of the household problem (the consumption Euler equation and the no arbitrage condition) and the laws of motion of capital and asset holdings close the consumer side of the model:

$$\frac{\dot{C}}{C}+\frac{\dot{P}_c}{P_c}=R-\rho, \tag{38}$$
$$r=(R+\delta-g_{P_x})P_x, \tag{39}$$
$$\dot{A}=RA+wL+rK-P_cC-P_xX, \tag{40}$$
$$\dot{K}=X-\delta K. \tag{41}$$

Turning to the firm side, the cost minimization problems of consumption and investment firms lead to

$$rk_c=wl_c\left(\frac{\alpha}{1-\alpha}\right), \tag{42}$$
$$rk_x=wl_x\left(\frac{\alpha}{1-\alpha}\right). \tag{43}$$

The innovation decisions of firms in both sectors and the entry decisions defined in equations (8) and (11) generate the following conditions

$$\chi_j\psi_j z_j^{\gamma/(1-\gamma)}=\frac{1}{1-\gamma}\chi_j b_j^{\gamma/(1-\gamma)},\quad j=c,x, \tag{44}$$
$$(R+\tau_c-b_c)w\chi_c\psi_c z_c^{\gamma/(1-\gamma)}=\pi_c-w\chi_c b_c^{1/(1-\gamma)}+\frac{\partial V(1,Z)}{\partial Z}\dot{Z}, \tag{45}$$
$$(R+\tau_x-b_x)w\chi_x\psi_x z_x^{\gamma/(1-\gamma)}=\pi_x-w\chi_x b_x^{1/(1-\gamma)}+\frac{\partial V(1,I)}{\partial I}\dot{I}, \tag{46}$$

Finally, the market clearing conditions for labor and capital close the model:

$$L = l_c+l_x+\sum_{j=c,x}\chi_j\psi_j z_j^{1/(1-\gamma)}+\sum_{j=c,x}\chi_j b_j^{1/(1-\gamma)}, \tag{47}$$
$$K = k_c+k_x. \tag{48}$$

B.3 Balanced Growth Path

To find the growth rates of the variables on the balanced growth path, I adopt the "guess-and-verify" method. Let $g_a\equiv\frac{\dot{a}}{a}$ denote the growth rate of any variable a on the balanced growth path. Let Y denote the GDP of the economy, $Y=C+P_xX$. Then the growth rate of consumption is equal to growth rate of investment expenditures, $g_C=g_I=g_{P_x}+g_X$. Using the income approach to GDP, $Y=rK+wL+RA-\dot{A}$, the growth rate of consumption is equal to growth rate of the wage rate, $g_C=g_w=g_r+g_K$. Since the price of consumption is normalized to 1, equation (35) implies that $g_{Q_c}=\alpha g_r+(1-\alpha)g_w$. Using the investment price formula in (37), $g_{P_x}+g_{Q_x}=\alpha g_r+(1-\alpha)g_w$. Imposing the no arbitrage condition in (39), the growth rate of rental rate of capital should be equal to the growth rate of the relative price of investment goods, $g_r=g_{P_x}$.

Combining the growth rate of consumption and investment price equations,

$$\begin{aligned}g_{Q_c} &= \alpha g_r + (1-\alpha) g_w, \\ g_{Q_x} &= (\alpha - 1) g_r + (1-\alpha) g_w,\end{aligned}$$

the growth rate of the wage and rental rates of capital can be solved as $g_w = g_{Q_c} + \frac{\alpha}{1-\alpha} g_{Q_x}$, and $g_r = g_{Q_c} - \frac{1-\alpha}{1-\alpha} g_{Q_x}$.

I now verify that the above growth rates are indeed the balanced growth path rates by appealing to the other equilibrium conditions. First, by equation (31), the growth rate of consumption should be equal to $g_C = \alpha g_K + g_{Q_c}$:

$$\alpha g_K + g_{Q_c} = \alpha(g_w - g_r) + g_{Q_c} = \frac{\alpha}{1-\alpha} g_{Q_x} + g_{Q_c},$$

where the right hand side of the equation is equal to the wage growth rate, which is equal to the consumption growth rate. It is straightforward to verify that the other equilibrium conditions are also satisfied.

B.4 Growth Rates of Average Quality Levels

In this economy innovations occur with a Poisson rate of τ. Hence, in a time interval of t, the probability of exactly m innovations occur is equal to $f(m,t) = \frac{(\tau t)^m \exp(-\tau t)}{m!}$. Assuming the law of large numbers holds, the probability of having exactly m innovations in a time interval is equal to measure of products that had m innovations in that interval [Grossman and Helpman (1991)]. Plugging this back into the average quality level equation,

$$\begin{aligned}Q_t &= \exp\left(\int_0^1 \ln q(\omega) d\omega\right) \\ &= \exp\left(\sum_{m=0}^{\infty} f(m,t) \ln \lambda^m\right) \\ &= \exp\left(\ln \lambda \sum_{m=0}^{\infty} f(m,t) m\right) \\ &= \exp\left(\ln(\lambda)\tau t\right),\end{aligned}$$

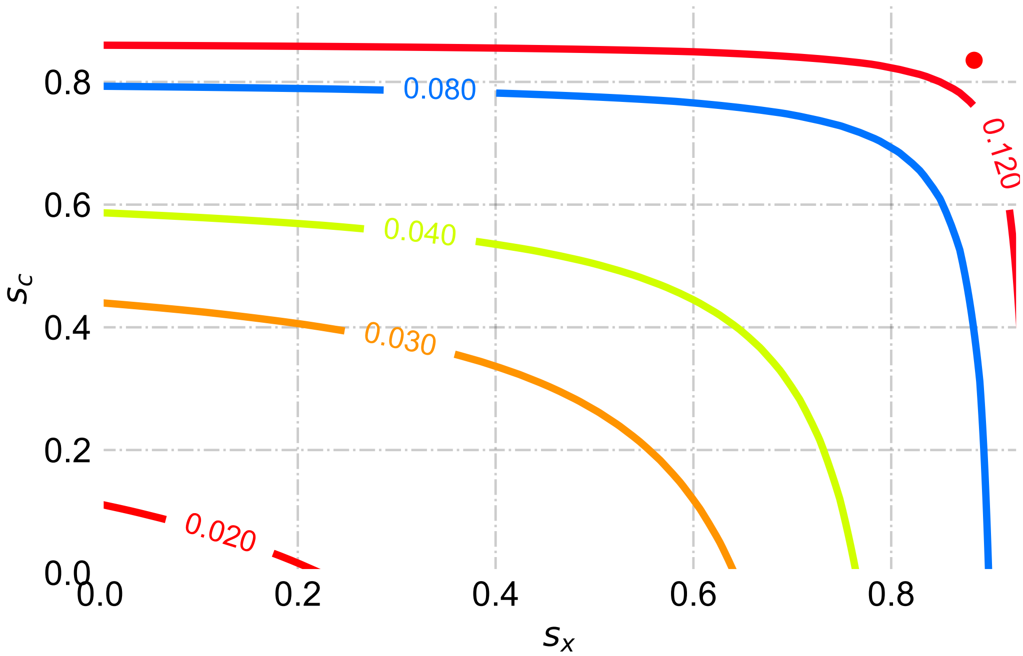
where the latter step is from the expectation of the Poisson distribution. The growth rate of average technology in each industry is then equal to

$$\frac{\dot{Q}_j}{Q_j} = \tau_j \ln \lambda_j, \quad j = c, x. \tag{49}$$

C R&D Expenditure as a Share of GDP

This section shows the total R&D expenditures by incumbent firms as a share of GDP at the balanced growth path.

Figure 8: Total Incumbent R&D Expenditure as a Share of GDP



Notes: Contour map of incumbent R&D expenditure as a share of GDP at the balanced growth path.

D Comparative Statics

As explained in the main text, subsidizing firm R&D leads to a trade-off between current and future consumption. An industry's location in the input-output chain impacts both the drop in current consumption and consumption growth rate. In Figures 9 and 10, I plot the percent changes in BGP consumption growth rate, production labor, and consumption equivalent welfare gain as a result of 10% increase in the R&D subsidy to investment goods producing sector for different values of α, the elasticity of consumption sector output with respect to capital. In these exercise, I keep the elasticity of investment sector output with respect to capital fixed at its benchmark value. To distinguish between the sector-specific α parameters, I denote by α_x the elasticity of investment sector output with respect to capital, so that α is related to the influence of investment sector in the input-output chain.

On the left panel, higher influence of the investment sector leads to higher consumption growth rates. Note that, at the BGP, $g_C = g_{Q_c} + \frac{\alpha}{1-\alpha_x} g_{Q_x}$. As α increases, an increase in rate of technological progress in investment sector, g_{Q_x}, generates higher consumption growth rates. The right panel of the Figure 9 shows a negative association between influence of the investment sector and the change in production labor as a result of R&D subsidy. Industries with low influence generate smaller reductions in production labor, and hence lower reductions in current period consumption.

The left panel suggests subsidizing higher influence industries as they lead to higher growth rates, while the right panel suggests subsidizing low influence industries as they lead to smaller reduction in current period consumption. The optimal subsidy depends on the which of these opposite factors dominates. Figure 10 plots the consumption–equivalent welfare gain of an extra 10% subsidy to the investment sector as a function of α. As α increases, so does the welfare gain resulting from an extra 10% subsidy to the investment sector. In other words, as the influence of an industry increases, the welfare gain associated with extra subsidy to that industry increases.

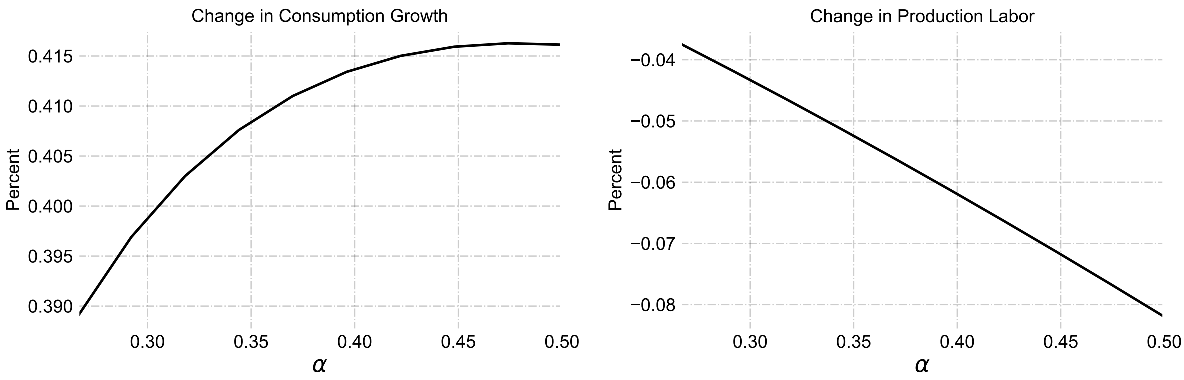
E Robustness

E.1 Alternative Calibration

Entrant innovation not only contributes productivity growth, but also presents a threat to incumbents by stealing production lines of incumbents. Because of this threat, incumbents discount future profit stream of a production line at a higher rate, $\rho + z$, than the social planner's ρ. In the original calibration, $(\rho + z_c)/\rho \approx 2.7$ and $(\rho + z_x)/\rho \approx 2.6$. In both sectors, the private discount rate is more than twice the social planner's discount rate. This large difference in discount rates leads to large under–investment in innovation in the competitive equilibrium, and underlies the high optimal subsidy.

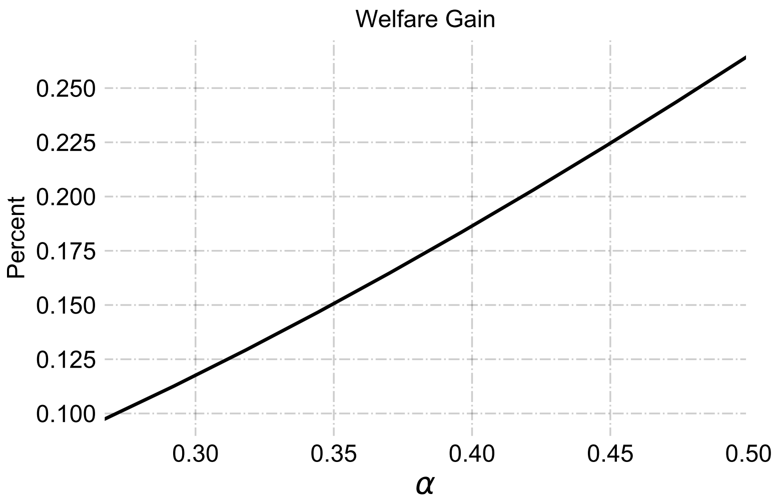
To check the robustness of my results, I re-calibrate with different targets. In the

Figure 9: Impact of 10% increase in R&D subsidy to investment good producers



Notes: The percent changes in BGP consumption growth rate and production labor as a result of 10% increase in the R&D subsidy to investment goods producing sector for different values of the elasticity of consumption sector output with respect to capital, α.

Figure 10: Welfare gain of 10% increase in R&D subsidy to investment good producers



Notes: Welfare gains in consumption equivalent terms as a result of 10% increase in the R&D subsidy to investment goods producing sector for different values of the elasticity of consumption sector output with respect to capital, α.

benchmark calibration, I relied on model's implications on job creation. However, the model also has implications on job destruction rates: the number of jobs created by entering firms is equal to the number of jobs destroyed by exiting firms; and the number of jobs created by expanding firms is equal to number of jobs destroyed by shrinking firms. Table 8 shows target moments calculated from the BDS.

Job destruction rate by death in the data is equal to the ratio of the number jobs destroyed by exiting establishments in an industry to employment in that industry. Job destruction rate by continuers in data is equal to the ratio of the number of jobs destroyed in establishments continuing their operations in an industry to total

Table 8: Alternative Targets

	Variable	Data	Model
Job destruction rate by death, consumption	z_c	0.044	0.044
Job destruction rate by death, investment	z_x	0.044	0.044
Job destruction rate of continuers, consumption	b_c	0.087	0.087
Job destruction rate of continuers, investment	b_x	0.092	0.092
GDP per capita growth rate	g_Y	0.015	0.015
Growth rate of investment good prices relative to consumption good prices	g_{P_x}	-0.028	-0.028
Labor's share of income		0.714	0.714

Notes: Target moments in an alternative calibration. This calibration relies on job destruction rates as opposed to job creation rates as in the benchmark calibration of Section 4.

employment in that industry.[27] After job destruction rates are calculated, I construct job destruction rates in the consumption and investment final goods sectors using the methodology described in detail in Appendix A.

In this alternative calibration, I target entry rates, z_c and z_x, to match job destruction rate by death in each industry. Similarly, I target incumbent firm innovation rates, b_c and b_x, to match job destruction rate by continuers. Entrant innovation rate targets in this calibration are lower than benchmark calibration, which is described in Section 4. The entrant innovation rate in the consumption sector falls to 4.4% from 5.2%, and the entrant innovation rate in the investment sector falls to 4.4% from 4.7%. Hence, the threat of entry on incumbents is lower both in the consumption and in the investment sector with this calibration. However, the private discount rates in both industries, $\rho + z$, are about 2.5 times the social planner discount rate, ρ, a substantial difference between private and social planner discount rates. The parameter values resulting from this calibration are reported in Table 9.

Table 9: Internally calibrated parameters, alternative calibration

	Parameter	Value
Quality ladder step size, investment	λ_x	1.26
Quality ladder step size, consumption	λ_c	1.03
R&D cost function parameter, investment	χ_x	7.83
R&D cost function parameter, consumption	χ_c	4.63
Entry cost function parameter, investment	ψ_x	3.03
Entry cost function parameter, consumption	ψ_c	2.73
Elasticity of output w.r.t capital	α	0.27

Under this calibration, the competitive economy still substantially under-invests in innovation. The social planner sets the consumption sector innovation rate to 23%, and the investment sector innovation rate to 29%, and the GDP growth rate to 3.1% in

[27] Here, I retain my identifying assumption that a firm in my model corresponds to an establishment in the data.

the long run. Transitioning from the competitive equilibrium balanced growth path to the social planner allocation results in a 20% welfare gain in consumption–equivalent terms. On the other hand, the government can use a subsidy system to generate a 18% welfare gain. In particular, the government approximates the social planner allocation with an 81% subsidy to the consumption incumbents and an 87% subsidy to the investment incumbents, all the while adjusting entrant innovation subsidy to correct for the congestion externality in the entry process.[28]

E.2 Further Robustness Checks on the Entry Rate

To further analyze the importance of entry rate, I match the entry rate over innovation rate in each industry to estimates of the contribution of entry to productivity growth from other papers by adjusting the entry cost parameters, ψ_c and ψ_x while keeping the other parameter values at their benchmark calibration values. Note that the estimates of the contribution of entry to productivity growth from the papers cited below are at the aggregate level, and I match the entrant share in each industry to these estimates. For each calibration, I calculate the social planner innovation rates at the balanced growth path and the welfare gain of moving from the competitive equilibrium to the social planner allocation. Table 10 shows the results of these exercises.

Table 10: Contribution of entry to sector productivity growth

Study	Entrant share	Social planner		
	In Productivity Growth	τ_c	τ_x	Welfare gain
Akcigit and Kerr (2018)	26.0%	0.25	0.3	18%
Garcia-Macia et al. (2016) (1976–1986)	19.1%	0.23	0.27	15%
Garcia-Macia et al. (2016) (2003 – 2013)	12.8 %	0.22	0.25	13%

Notes: Comparative statics on the cost–of–entry parameters, ψ_c and ψ_x, set to match the contribution of entry to productivity growth in each sector to estimates from other studies. Other parameters stay at the benchmark calibration values. τ_c and τ_x are balanced growth path innovation rates in the social planner economy and "Welfare gain" is the welfare gain associated with moving from the market economy to the social planner problem.

Akcigit and Kerr (2018) estimate the contribution of entry to productivity growth to be about 26 percent. Atkeson and Burstein (2019) use this estimate in their calibration. A lower entry rate in each industry means a lower private discount rate of future profit streams from successful innovation, and hence a smaller distortion. In particular, the lower entry rate diminishes the inter-temporal spillover effect, thereby reducing the socially optimal innovation rates relative to the benchmark calibration. Under this calibration, the social planner generates an 18 percent welfare gain.

[28]Recall that I restrain the government from correcting distortions in the Euler equation that result from monopoly pricing power of the investment good producers.

Garcia-Macia et al. (2016) estimate the contribution of entry to productivity growth for two time periods, 1976-1986 and 2003-2013. Calibrating my model to their lower estimate, 12.8 percent, reduces the socially optimal innovation rates and the welfare gain even further. This exercise highlights the importance of selecting the appropriate entry rate target. The lower the entry rate, the lower the distortion, and hence the lower the welfare gain from moving to the social planner allocation. Despite the sensitivity of the inter-temporal spillover effect to the targeted entry rate, the remaining externalities generate under–investment in innovation regardless of the particular target, thus leaving room for welfare–improving subsidy systems.

E.3 Subsidizing only the Incumbent Firms

I conduct an optimal subsidy analysis in which only the incumbent firms are subsidized, with the subsidies to entering firms kept at their benchmark calibration levels ($s_x^e = s_c^e = -1.25$). Table 11 shows the results. In this incumbent–only exercise, the optimal innovation subsidy to incumbents in the consumption sector is 75 percent, 8 percentage points lower than the optimal subsidy when entrants are also subsidized. Similarly, the optimal R&D subsidy to investment sector incumbents is 7 percentage points lower than the optimal subsidy when entry is subsidized. Table 11 shows that entry rates in the incumbent-only exercise decline from their BGP levels, thereby diminishing the inter-temporal spillover effect in each industry (since the private discount factor, $\rho + z$, approaches the social discount factor, ρ). In a sense, the incumbent–only exercise does not increase the inter-temporal spillover effect. The resulting optimal subsidy rates to incumbents are still large, but lower than the optimal subsidy rates when inter-temporal spillover effect is allowed to increase.

Table 11: Optimal R&D subsidy

Exercise	s_c	s_x	z_c	z_x	b_c	b_x	g_C	Welfare gain
Initial BGP	0.100	0.100	0.052	0.047	0.097	0.099	0.015	0.000
Optimal	0.835	0.884	0.095	0.105	0.178	0.221	0.032	0.203
Incumbent only	0.753	0.812	0.041	0.037	0.182	0.218	0.025	0.113

Notes: "Initial BGP" corresponds to the balanced growth path of the benchmark calibration; "Optimal" to the balanced growth path of the economy with incumbents and entrants both optimally subsidized; and "Incumbent only" to the balanced growth path of the economy when only incumbents are optimally subsidized, with entry subsidies at their benchmark calibration levels.

E.4 Curvature of R&D Cost Function

This section contains a comparative static exercise with respect to the curvature of the R&D cost functions. In the model, the cost of innovation for incumbents in industry i is $\chi_i b_i^\gamma$, where b_i is the innovation rate. In the benchmark calibration, the curvature of the R&D cost function, γ, is equal to 2.5. In the proposed exercise, I increase γ to 3.5,[29] and calculate the optimal R&D subsidy and the associated welfare gain. The optimal R&D subsidy rates are 78% to the consumption sector and 83% to the investment sector, and the welfare gain of this policy is 8%. Although the optimal subsidies are still high, the associated welfare gain is substantially lower than that in the benchmark model. A more convex R&D cost function requires a larger reduction in production labor to achieve any given productivity growth rate. As a result, the associated welfare gain of innovation policy is smaller. This policy results in larger BGP innovation rates in the consumption and investment sectors (from 14% and 14% respectively, to 21% and 22%). A more convex R&D cost function implies a higher marginal cost associated with increasing the innovation rate, and thus narrows the gap between the competitive equilibrium and socially optimal innovation rates.

F Calibration with patenting/R&D expense data

In the main body of the paper, I classify any activity that leads to job creation as innovation. I also target job creation by entering establishments and job creation of expanding establishments when estimating the model. In this section, I calibrate my model using patenting and R&D expense data. In particular, I target the following moments:

1. *The relative innovation rate in the investment sector,* τ_x/τ_c.

   I proxy the innovation rate with the *patenting rate*, the ratio of the number of patents granted to an industry in a year to the total number of establishment in that industry. Using NBER Patent Database (Hall et al. (2001)), I first calculate the total number of patents granted by application year for each USPC technology class. Then, using the probabilistic crosswalk of Lybbert and Zolas (2014), I link USPC technology classes with 2-digit NAICS industries. I then merge the patenting data with BDS, which results in a sample period of 1997-2000.

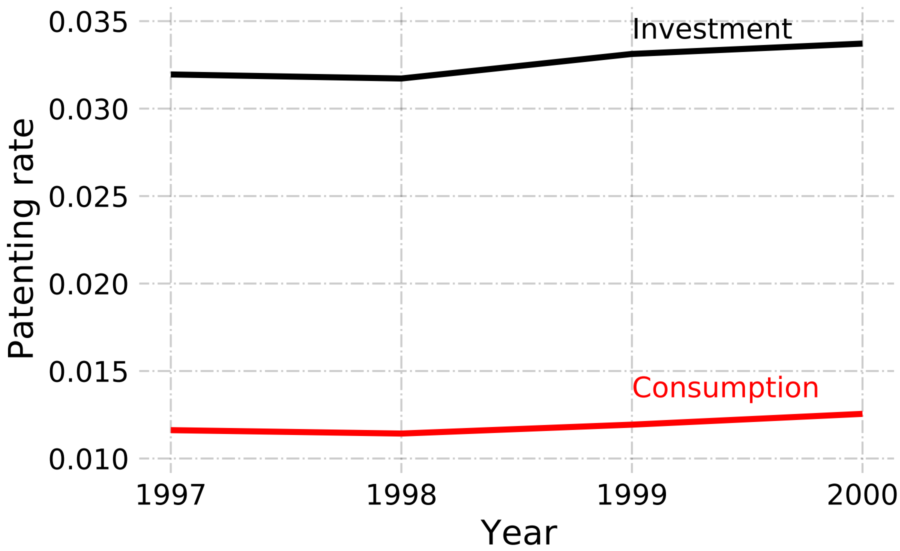
   Having calculated the patenting rate in each NAICS industry, I calculate the patenting rate in the consumption and investment sectors as weighted averages of industry patenting rates as described in Appendix A. The resulting patenting rates are plotted in Figure 11. The patenting rate in the investment sector is about 2.76 times the patenting rate in the consumption sector. Therefore, I target τ_x/τ_c to be equal to 2.76 in the calibration.

2. *The relative R&D intensity in the investment sector,* $\frac{\chi_x b_x^\gamma}{\chi_c b_c^\gamma} \frac{l_{p,c}}{l_{p,x}}$.

   Define an industry's *R&D intensity* as the ratio of domestic R&D expenses in that industry to total employment in that industry. I gather data on industry–level R&D expenditures from the Business Research and Development and Innovation Survey (BRDIS) from 2011 to 2016 (NSF (2019)). I combine domestic R&D

[29] I also adjust χ_i to keep the total R&D labor required to achieve the benchmark calibration innovation rates constant.

Figure 11: Patenting Rate of Industries



Notes: The patenting rate is calculated as the ratio of the number of patents granted to an industry in a year to the total number of establishment in that industry. Industry level patenting rates in the data are converted into consumption/investment sector level by taking weighted averages of patenting rate in each industry.

expense data[30] in certain NAICS industries with employment information from BDS. I then calculate the R&D intensities of the consumption and investment sectors as weighted averages of 2-digit NAICS industries. In my sample, the R&D intensity of the investment sector is about 2.58 times of the consumption sector. Therefore, I target $\frac{\chi_x b_x^\gamma}{\chi_c b_c^\gamma} \frac{l_{p,c}}{l_{p,x}}$ to be equal to 2.58.

One caveat concerning the R&D expenses data is that publicly available tabulations of the BRDIS does not contain information about every NAICS super–industry. Some industries are pooled together and reported as one observation. Therefore, I keep only the industries in BRDIS data that have clear and obvious links to the BDS industries.[31,32] These remaining industries cover approximately 58 percent of the consumption sector and 77 percent of the investment sector. In computing weighted average of industry R&D intensities to construct consumption/investment sector intensities, there are two options: (i) re-normalize the weights to sum to unity in both the consumption and investment sectors, (ii)

[30] R&D expense includes only domestic R&D paid and performed by firms.

[31] The industry coding in BDS follows the SIC classification. Please refer to Appendix A on the crosswalk between SIC industries in BDS to NAICS industries.

[32] The following dictionary relates NAICS industries reported in BRDIS and their corresponding BDS matches: Industry 31–33 in BRDIS is matched to Manufacturing, no match for '21–23, 42–81' in BDS, 21 in BRDIS is matched to Mining and logging in BDS, 22 in BRDIS to Utilities in BDS, 42 in BRDIS to Wholesale trade in BDS, 454111–12 in BRDIS to Retail trade in BDS, 48–49 in BRDIS to Transportation and warehousing in BDS, 51 in BRDIS to Information in BDS, 54 in BRDIS to Professional and business services in BDS, 621–23 in BRDIS to Education and health services in BDS, no match for '23, 44–45 (excluding 45411112), 55–56, 624, 71–72, 81' in BDS.

use the weights as they are. The first option artificially increases R&D intensity of the consumption sector since high R&D intensity industries receive higher consumption shares after normalization. I pursue the second option because the industries that are pooled together (labeled as "other non-manufacturing" in the BRDIS) have a very low R&D/sales ratio, approximately 0.3%, as opposed to above-4% in the manufacturing industries. Therefore, pursuing option (ii) and treating "non-manufacturing" industry in the BRDIS as though it did not conduct any R&D would not affect the R&D intensity ratios considerably.

3. *The ratio of entrant-to-incumbent innovation rates, z_i/b_i.*

   The two targets above do not distinguish between entrant and incumbent innovation. In this calibration, I assume the ratio of the innovation rate of entrants to that among incumbents is equal to their job creation rate ratios. Therefore, I target z_x/b_x to be equal to 0.475 and z_c/b_c to be equal to 0.534 as in the benchmark calibration.

4. *Relative employment at investment-sector establishments, $l_{p,x}/l_{p,c}$.*

   In contrast to the benchmark calibration, the average product line sizes in the two industries are relevant. Since the relative R&D intensity in the investment sector is a function of employment at a product line in each industry, I introduce an additional target in this calibration: $l_{p,x}/l_{p,c}$. In BDS, the mean number of employees of establishments in the investment sector is 15 percent higher than that in consumption sector establishments. I therefore target $l_{p,x}/l_{p,c}$ to be equal to 1.15. To achieve such target, I allow the consumption goods to be located in the $[0, N]$ interval, for some N to be estimated, whereas the investment sector goods are still located in the $[0, 1]$ interval.

Table 12 reports the data and model moments that are targeted in the estimation. It is evident that the model does well in reaching the targets. The resulting parameter estimates are reported in Table 13.

Table 12: Targeted data and model moments

	R&D intensity ratio	$\frac{\tau_x}{\tau_c}$	g_C	g_r	Labor share	$\frac{l_{p,x}}{l_{p,c}}$	$\frac{z_c}{b_c}$	$\frac{z_x}{b_x}$
Data	2.58	2.76	0.015	-0.028	0.714	1.15	0.534	0.475
Model	2.58	2.76	0.015	-0.028	0.714	1.15	0.534	0.475

Notes: The data moments which are targeted in the calibration of Section F and their corresponding values in the model. "R&D intensity" reports the ratio of R&D expenses to employment in an industry. "R&D intensity ratio" reports the R&D intensity in the investment sector relative to the R&D intensity in the consumption sector.

Table 13 shows some stark differences in the parameter estimates relative to the benchmark calibration reported in Table 4. First, the innovative steps, λ_x and λ_c, are dramatically higher than the benchmark calibration, with the innovative step in the consumption sector particularly large. Second, the consumption sector cost of innovation, χ_c, is much higher than the benchmark calibration. Third, the output elasticity of capital is lower than the benchmark calibration.

Table 13: Parameter estimates

	λ	χ	ψ	Size	α
Consumption	1.316	43.788	0.036	17.293	0.122
Investment	1.483	9.313	3.051	1.000	0.122

Notes: Estimated parameter values from the calibration exercise.

Since there is no closed form solution of the mapping from data moment targets to parameter values, one cannot attribute a change in one particular parameter estimate to a particular moment. However, I can still describe the impact of the new calibration targets on parameter estimates. First, the benchmark calibration targets τ_x/τ_c to be approximately equal to 1. In contrast, the alternative calibration targets τ_x/τ_c to equal to 2.77, while holding the GDP growth rate target at 1.5 percent. Therefore, a lower innovation rate in the consumption sector requires a higher innovative step in the consumption sector.[33] Second, in this calibration, the relative R&D intensity ratio (2.58) is close to the relative innovation rate (2.76). Moreover, the cost of innovation, $\chi_i b_i^\gamma$, is convex in the innovation rate when $\gamma = 2.5$. The scale parameter in the cost innovation in the consumption sector, χ_c, therefore needs to be large if the relative R&D intensity ratio is to be close to the relative innovation rate. Third, larger innovative steps increase industries' profit shares, which mechanically diminishes the labor share. To compensate for the reduction in labor share resulting from higher markups and achieve the targeted labor's share of income, the calibrated capital elasticity of output must fall. Key equilibrium values of the balanced growth path are reported in Table 14.

Table 14: Industry level equilibrium values

	τ	b	z	l_p
Consumption	0.035	0.023	0.012	0.048
Investment	0.097	0.065	0.031	0.056

Notes: Equilibrium values of select variables in the balanced growth path of the model for the consumption and the investment goods sectors.

Having calibrated the model, I now solve for the unconstrained– and constrained–optimum innovation subsidy variants of the economy.

[33]Recall that the consumption sector growth rate is $g_C = \tau_c \ln \lambda_c + \alpha/(1-\alpha)\tau_x \ln \lambda_x$

Optimal Innovation Subsidies

The government subsidizes consumption– and investment–sector incumbent R&D spending at 32% and 84% respectively; see Table 15.[34] The optimal subsidy increases the innovation rate in the consumption sector from 3.5% in the competitive equilibrium (CE) to 3.8%, a mild increase. In contrast, the optimal subsidy generates a substantial increase in the investment-sector innovation rate, from 9.7% to to 20.6%. These increased innovation rates hike consumption growth to 2.2% in the long run, at a cost of 6.8% of GDP.[35] Overall, the optimal subsidy generates a 6% welfare gain.

Table 15: Optimal R&D subsidy

	s_c	s_x	τ_c	τ_x	g_C	Welfare	R&D subsidy/GDP
CE	0.100	0.100	0.035	0.097	0.015	0.000	0.005
Optimal	0.325	0.840	0.038	0.206	0.022	0.061	0.068

Notes: CE refers to the competitive equilibrium. Subsidy rates and equilibrium values of variables in the balanced growth path under the competitive equilibrium and optimal subsidy system.

The optimal subsidy to the consumption sector under this calibration (32%) is substantially lower than the 84% optimal subsidy to the consumption sector under the benchmark calibration in the main text. In contrast, the optimal subsidy to the investment sector does not vary significantly across calibrations (84% under this calibration and 88% under the benchmark calibration). Finally, the 6% welfare gain generated by the optimal subsidy under this calibration is also dramatically lower than the 20% welfare gain resulting from optimal subsidy under the benchmark calibration.

Constrained–Optimum Subsidies

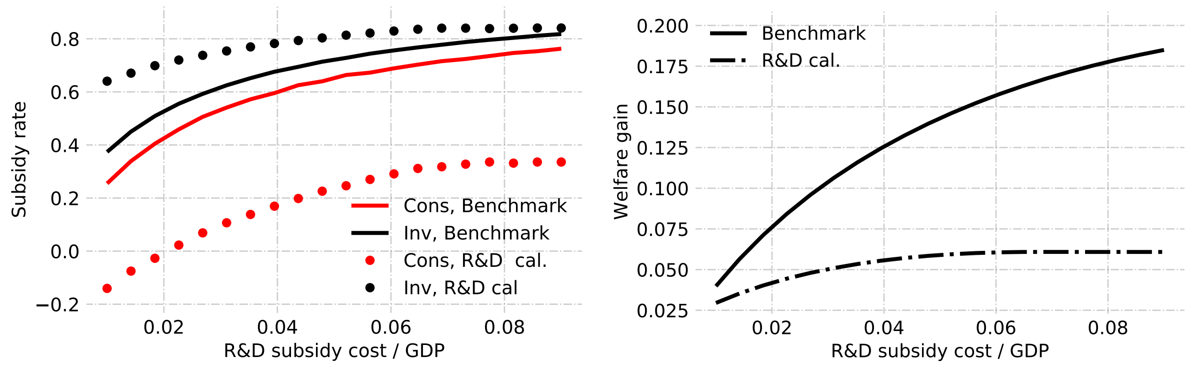
As in section 6.1, I find the optimal innovation to incumbent firms in each industry when the total subsidy distributed to incumbent firms in the economy is limited to a given fraction of GDP. Figure 12 shows the constrained optimum subsidy rates and the associated welfare gain in consumption equivalent terms. To compare the results with the benchmark economy, the figure also shows the results of the corresponding analysis with the benchmark calibration.

Three results emerge. First, with a low R&D subsidy budget, it is optimal to tax the consumption sector and subsidize the investment sector R&D. Second, under this calibration, the (constrained–optimum) subsidy to the consumption sector is always smaller than the corresponding subsidy under the benchmark calibration, whereas the subsidy to the investment sector is always higher than its benchmark calibration counterpart. Finally, the welfare gain under this calibration is always lower than that under the benchmark calibration.

[34] As before, for any incumbent subsidy s_i, entrants are subsidized at a rate $1 - (1 - s_i) * \gamma$ to maintain the optimum allocation to innovative resources between entrants and incumbents.

[35] As before, R&D subsidy cost includes only subsidies to the incumbent firms.

Figure 12: Constrained–Optimum R&D Subsidy Rates and Welfare Gains



Notes: Optimal R&D subsidies to sectors with limited transfer budget (left panel) and the corresponding welfare gain (right panel), comparing constrained–optimum subsidies under this calibration and under the benchmark calibration described in section 4. "Cons, R&D cal." corresponds to the constrained–optimum subsidy to the consumption sector under this calibration; "Inv, R&D. cal." to the constrained–optimum subsidy to the investment sector under this calibration; "Cons, Benchmark" to the constrained–optimum subsidy to the consumption sector under the benchmark calibration; and "Inv, Benchmark" to the constrained–optimum subsidy to the investment sector under this calibration.

Explaining the sectoral disparity in optimal subsidies

To understand why it is optimal to subsidize consumption sector R&D at considerably lower rates than investment sector R&D, I turn to the social planner problem. The following equation characterizes the allocation of the innovative activities between the sectors in the social planner economy:

$$\frac{N\gamma\chi_c b_c^{\gamma-1} F_{L_c}(Q_c, K_c, L_c)}{\gamma\chi_x b_x^{\gamma-1} F_{L_x}(Q_x, K_x, L_x)} = \frac{\ln\lambda_c}{\ln\lambda_x}\frac{F(Q_c, K_c, L_c)}{F(Q_x, K_x, L_x)}, \tag{50}$$

where L_i is total production labor, K_i is total capital stock, and Q_i is aggregate productivity in sector $i = c, x$, and $F(\cdot)$ is the aggregate production function. The left hand side of the equation is the ratio of the marginal costs of innovation in the industries, whereas the right hand side is the ratio of the marginal benefits of innovation in the industries.[36] An increase in innovation reduces production by $N\gamma\chi_i b_i^{\gamma-1} F_{L_i}(Q_i, K_i, L_i)$ in industry i, whereas the an increase in the growth rate of the economy increases production by $\ln\lambda_i F(Q_i, K_i, L_i)$.

Rearranging (50), the allocation of innovative resources between sectors, which is proportional to b_c/b_x, is determined by the three ratios.

[36]The left hand side of equation (50) invokes the envelope theorem: the marginal cost of total innovation is equal to the marginal cost of incumbent innovation at optimum.

The relative cost of innovation in the consumption sector, $\frac{\chi_c}{\chi_x}$. This term is higher in the current calibration than in the benchmark calibration so that, all else equal, the optimal consumption sector innovation rate in this calibration would be lower than that in the benchmark calibration.

The relative innovation-induced marginal increase in the growth of average quality in the consumption sector, $\frac{\ln \lambda_c}{\ln \lambda_x}$. This term is larger in the current calibration than in the benchmark calibration, so that, all else equal, the optimal consumption sector innovation rate in this calibration would be higher than that in the benchmark calibration. However, this effect roughly offsets 70% of the increase in the relative cost of innovation in the consumption sector, resulting in similar optimal innovation ratios if one focuses only on these two factors.

The relative average production line size (employment) in the consumption sector, $\frac{L_c/N}{L_x}$.[37] This term is considerably lower in the current calibration than the benchmark calibration because the current calibration targets the relative average industry size. Since the average size of consumption sector production line relative to the investment sector is considerably lower in the current calibration than in the benchmark calibration, it is optimal to allocate considerably fewer resources to the consumption sector. Although the social planner chooses $\frac{L_c/N}{L_x} = \frac{l_c}{l_x}$, the calibration $\frac{l_c}{l_x}$ is targeted to be equal to 1/1.15 so that, by construction, total production in the consumption sector is allocated to larger number of firms in order the keep average production line sizes in line with the data.

Why does the relative number of production lines have a stark implication on the results? Although this ratio has no effect on production, which exhibits constant returns to scale, it matters for innovation because the growth rate of the average productivity (quality) of industries depends on the average innovation rate in the economy. Spreading a fixed amount of innovative resources over a larger measure of production lines therefore reduces the economy's growth. In the current calibration, the measure of production lines in the consumption sector is considerably larger than the investment sector. Therefore, the consumption sector requires more innovative resources to achieve a high rate of technological progress in the current calibration, and the social planner does not choose as high a consumption–sector innovation rate as in the benchmark calibration.

Since the conditional optimal innovation rates in the consumption sector are lower than the benchmark calibration, the subsidy rates required to decentralize such innovation rates are also lower. Consequently, the conditional optimal subsidy to the consumption sector under the current calibration is lower than the benchmark calibration.

[37] In equilibrium, $L_c/L_x = (1 - A)/A$, where $A \equiv \alpha\left(\delta + \frac{\tau_x \ln \lambda_x}{1-\alpha}\right)\left(\rho + \delta + \frac{\tau_x \ln \lambda_x}{1-\alpha}\right)^{-1}$, so that $\frac{F(Q_c,K_c,L_c)/(NF_{L_c}(Q_c,K_c,L_c))}{F(Q_x,K_x,L_x)/F_{L_x}(Q_x,K_x,L_x)} = \frac{L_c/N}{L_x}$.

G Elastic Labor Supply

The main body of the paper assumes inelastically supplied labor. However, large subsidies to R&D may alter equilibrium labor through two channels. First, higher wages resulting from increased demand for labor is likely to increase equilibrium labor. Second, higher *labor* tax rates required to finance R&D subsidies reduce after-tax wages, and are likely to depress labor supply. Depending on the relative strengths of these two forces, R&D subsidies financed by distortionary labor taxes may alter the optimal subsidy. To account for equilibrium labor changes resulting from increased R&D subsidies, I repeat the optimal and constrained–optimum subsidy analyses with elastic labor supply. In this section, the instantaneous utility function is

$$\ln(C) - \xi \frac{L^{1+1/\varphi}}{1+1/\varphi},$$

where L is labor supply, φ is the Frisch elasticity of labor supply, and $\xi > 0$. Suppose the government finances incumbent R&D subsidies with distortionary (linear) labor taxes, and taxes or subsidizes entry accordingly so that innovative resources are allocated efficiently between entrants and incumbents. To highlight the role of incumbent R&D subsidies, suppose that the government finances entry subsidies with lump-sum transfers. Finally, the government maintains a balanced budget:

$$\begin{aligned} t_l w L &= s_{i,c}\chi_c b_c^\gamma + s_{i,x}\chi_x b_x^\gamma, \\ T &= s_{e,c}\psi_c\chi_c z_c^\gamma + s_{e,x}\psi_x\chi_x z_x^\gamma, \end{aligned}$$

where t_l is the rate of labor income taxation, and T is the amount of lump-sum tax/transfer.

I calibrate and solve the model with two different values of Frisch elasticity (φ): 0.5 and 2, the former closer to estimates from micro data[38] and the latter close to macroeconomic estimates. In either case, I calibrate the model to match the target moments as in the benchmark economy and set labor supply equal to 1.[39] Having calibrating the model, I perform the optimal and constrained-optimal subsidy analyses.

Optimal Innovation Subsidies

Table 16 reports the unconstrained optimal subsidy analysis. A few results emerge. First, the optimal innovation subsidy increases household labor supply at the new balanced growth path, relative to the benchmark. The higher wages resulting from increased demand for R&D labor outweigh the negative impacts of the higher taxes required to finance R&D subsidies. With a low Frisch elasticity, labor supply increases 4.6%, whereas with a high Frisch elasticity, labor supply increases 10.5%. In the market economy, equilibrium labor is lower than the socially optimal level. Because

[38] See Keane and Rogerson (2012) for a survey of micro and macro labor supply elasticities.

[39] I set $\xi = 0.891$ for both values of φ. Other model parameters remain the same as the benchmark calibration

the R&D subsidy increases equilibrium labor from its sub-optimal baseline level, an even larger subsidy is necessary to achieve this goal with distortionary taxation.

Second, the long run growth rate of the economy is greater when labor supply is elastic. Naturally, with greater labor supply, the economy can allocate more resources to R&D, which, in turn generates a higher growth rate in the long-run. Third, the optimal subsidy generates higher welfare gains with elastic labor supply compared to the inelastic labor supply. The more elastic is the labor supply, the higher are the welfare gain and the tax rate required to finance the subsidy.

Table 16: Optimal Subsidy with Elastic Labor Supply

φ	s_c^*	s_x^*	Labor	g_C	Tax rate	Subsidy/GDP	Welfare gain
Inelastic	0.835	0.884	1.000	0.032	N/A	0.155	0.203
0.5	0.847	0.892	1.046	0.033	0.174	0.170	0.217
2.0	0.860	0.902	1.105	0.035	0.189	0.191	0.234

Notes: Balanced growth path values of some variables with optimal subsidy rates, s_c^* and s_x^*, and their associated welfare gains. “Inelastic” refers to the optimal subsidy applied to the benchmark economy described in Section 4. “Tax rates” refers to the labor tax rate required to finance R&D subsidies. “Subsidy/GDP” refers to the cost of R&D subsidies as a share of GDP.

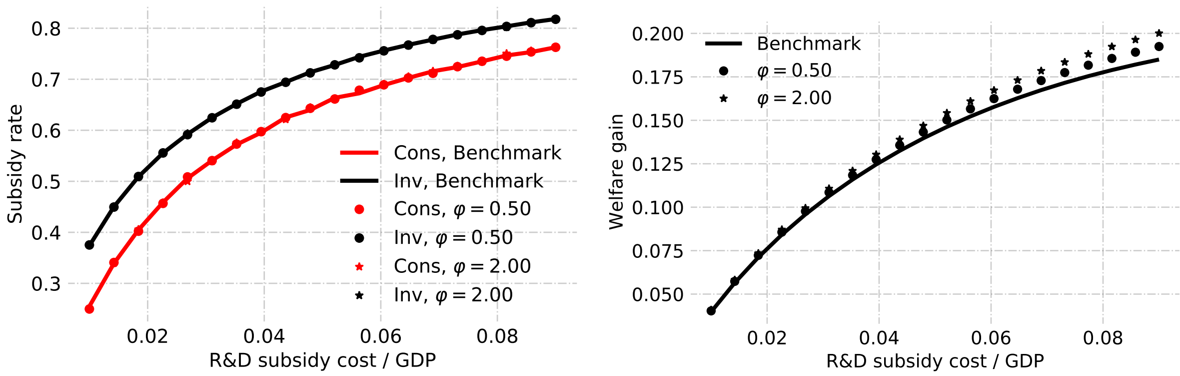
Constrained-Optimum Subsidies

Figure 13 depicts the constrained–optimum subsidy analysis with elastic labor supply. As in the benchmark model, the R&D subsidies in both industries are increasing with to the total subsidy budget; further, it is always optimal to subsidize the investment sector R&D at a higher rate than the consumption sector. More strikingly, the conditional optimal subsidy rates does not vary noticeably with the Frisch elasticity of labor supply, as seen in the left panel of the figure. However, the associated welfare gain increases with the Frisch elasticity, as shown in the right panel of the figure. With elastic labor supply, the higher the subsidy rate, the larger is the increase in labor supply. The additional labor supply is absorbed in R%D and in goods production, resulting in larger welfare gains.

Without a closed form solution, it is impossible to determine whether the conditional optimal subsidy independent of Frisch elasticity of labor supply. However, a simplified version of the model demonstrates that the optimal subsidy varies, albeit marginally, with the Frisch elasticity of labor supply. Consider a model with only one sector, no capital in the production function, and where only entrants innovate with a cost function χz^γ. For a given subsidy rate, s, the equilibrium R&D subsidy cost as a fraction of GDP is

$$\frac{s\chi z^\gamma}{\lambda l_p} = \frac{\lambda - 1}{\lambda}\left[\frac{(1-s)\gamma}{s}\left(\frac{\rho}{z}+1\right)\right]^{-1},$$

Figure 13: Constrained–Optimum R&D Subsidy



Notes: Optimal R&D subsidies to sectors under limited transfer budget and associated welfare gain, comparing elastic and inelastic (benchmark) labor supply models.

where γ, ρ, λ are identical across models with differing Frisch elasticites. Note that the Frisch elasticity affects the above ratio only through the equilibrium entrant innovation rate, z. However, the resulting differences in cost of R&D subsidy, relative to GDP, are minimal – barely distinguishable to the human eye graphs like Figure 13.

In summary, this analysis yields three takeaways. First, the cost of R&D subsidy, relative to GDP, is not identical but very similar across models with differing Frisch elasticities. Second, the cost of R&D, relative to GDP, is increasing with the subsidy rate (at least in the region where the constraint binds). Finally, welfare gains are increasing in the subsidy rate (again at the region where the constraint binds). Given the last two observations, a welfare maximizing R&D subsidy would utilize all the available subsidy budget. Adding the first observation, the associated subsidy that utilizes the whole budget would vary minimally with the Frisch elasticity. Therefore, the constrained optimal subsidies are fairly insensitive, and the associated welfare gains more sensitive, to changes in the Frisch labor supply elasticity.

H Multi-sector Model

Consider an extension of my model with n-sectors and intermediate inputs. Gross output production is Cobb-Douglas in capital, labor and intermediate inputs:

$$y_{fi} = \left(k_{fi}^{\alpha} l_{fi}^{1-\alpha}\right)^{\sigma} \left(\prod_j m_{fij}^{\omega_{ij}}\right)^{1-\sigma},$$

where y_{fi} is gross output of product f in industry i, k_{fi} and l_{fi} are capital and labor used in production of f, and m_{fij} is the amount of intermediate input from industry j used in product f in industry i. A competitive sector purchases firm output and produces industry output with production function $y_i = \exp\left(\int_0^1 \ln\left(q_{fi} y_{fi}\right) df\right)$, where q_{fi} is the quality of product f in industry i. Competitive sectors use the output of each industry to produce final consumption and investment goods with the following production functions:

$$C = \prod_j c_j^{\zeta_j}, \text{ and } I = \prod_j i_j^{\phi_j}.$$

The remaining parts of the model are identical to the two-sector model described in Section 2. A representative consumer inelastically supplies one unit of labor and maximizes the discounted sum of logarithmic utility stemming from consumption by choosing consumption and investment. Firms, defined as combinations of different product lines, produce and invest in R&D. In the model, a firm owns productions lines from only one sector. As a result of Bertrand competition and unit elastic demand for each product within a sector, the equilibrium output of sector i is equal to $y_i = \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{\sigma} \left(\prod_j m_{ij}^{\omega_{ij}}\right)^{1-\sigma} Q_i$, where $Q_i \equiv \exp\left(\int_0^1 \ln q_{fi} df\right)$.

Let M be the collection of intermediate inputs supplied to industry i from industry j:

$$\underset{n\times n}{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & \dots & m_{1n} \\ m_{21} & m_{22} & m_{23} & \dots & m_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & m_{n3} & \dots & m_{nn} \end{bmatrix}.$$

Collecting model parameters in Ω, Z, Φ as

$$\underset{n\times n}{\Omega} = \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} & \dots & \omega_{1n} \\ \omega_{21} & \omega_{22} & \omega_{23} & \dots & \omega_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega_{n1} & \omega_{n2} & \omega_{n3} & \dots & \omega_{nn} \end{bmatrix}, \quad \underset{n\times 1}{Z} = \begin{bmatrix} \zeta_1 \\ \vdots \\ \zeta_n \end{bmatrix}, \quad \underset{n\times 1}{\Phi} = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_n \end{bmatrix},$$

the growth rate of industry level output, consumption, and investment can be written as

$$g_y = \mathbf{B} \begin{bmatrix} g_{Q_1} \\ \vdots \\ g_{Q_n} \end{bmatrix}, \quad g_C = Z \times g_y^T, \quad g_I = \Phi \times g_y^T,$$

where

$$\mathbf{B} \equiv \left(I_n - \alpha\sigma \begin{bmatrix} \phi_1 \cdots \phi_n \\ \vdots \dots \vdots \\ \phi_1 \cdots \phi_n \end{bmatrix} - (1-\sigma)\Omega \right)^{-1}.$$

H.1 Constrained Optimum

Consider an exercise such that the social planner allocates (only) innovative resources across sectors while holding the total innovative input at the competitive equilibrium level. Since the social planner does not alter the production decisions, the optimal allocation of innovative resources maximizes the growth rate of the economy while holding R&D labor constant:

$$\max_{\{\tau_i\}_{i=0,..,n}} D\,\mathbf{g_Q},$$

subject to $\sum_{i=0}^{n} C(\tau_i) = L_{R\&D}$, where $D = \mathbf{Z}^T\mathbf{B}$, and $C(\tau_i) = A_i\tau_i^{\gamma}$ is the labor cost when the market allocates a total innovation rate $\tau_i = z_i + b_i$ in an industry, with entrant innovation z_i and incumbent innovation b_i, where

$$A_i \equiv \chi_i \left(\frac{1}{\left(\frac{(1-s_i)\gamma}{(1-s_e)\psi_i}\right)^{1/(\gamma-1)} + 1} \right)^{\gamma} \left(\psi_i \left(\frac{(1-s_i)\gamma}{(1-s_e)\psi_i} \right)^{\gamma/(\gamma-1)} + 1 \right).$$

The growth-rate-maximizing innovation rate is

$$\tau_i = \left(\frac{D_i \ln \lambda_i}{A_i \gamma} \right)^{1/(\gamma-1)} \left(\frac{L_{R\&D}}{\sum A_i \left(\frac{D_i \ln \lambda_i}{A_i \gamma_i} \right)^{\gamma/(\gamma-1)}} \right)^{1/\gamma}.$$

In an environment with identical innovation functions (both innovative steps and cost of innovation) across sectors, the ratio of innovation rates in sectors i and j simplifies to

$$\frac{\tau_i}{\tau_j} = \left(\frac{D_i}{D_j} \right)^{1/\gamma-1}.$$

H.1.1 Vertical Economy

Consider an example vertical economy with 10 sectors, where sector $i = 0, 1, \ldots, 9$ uses labor and intermediate input from sector $i+1$ in production, and industry 9, the furthest upstream, uses only labor in production. There is no capital in production, that is, $\alpha = 0$. In this specific example, $D_i/D_j = (1-\sigma)^{i-j}$, therefore the relative innovation rate in sector i is

$$\frac{\tau_i}{\tau_j} = \left((1-\sigma)^{i-j} \right)^{1/(\gamma-1)}.$$

Contrast constrained optimal allocation to competitive equilibrium allocation:

$$\frac{C'(\tau_i)(\rho + \tau_i - b_i) + c(b_i)}{C'(\tau_j)(\rho + \tau_j - b_j) + c(b_j)} = \frac{\pi_i}{\pi_j} = \left(\frac{1-\sigma}{\lambda} \right)^{i-j}.$$

Various distortions in the competitive equilibrium lead sector innovation rates to diverge from the constrained optimum.

Monopoly distortion. First, the existence of market power pushes sectoral profit ratios toward downstream industries, beyond their influence on the final consumption: $((1-\sigma)/\lambda)^{i-j} > (1-\sigma)^{i-j}$ when $i < j$, i.e., when i is more downstream.

Inter-temporal spillovers. Second, the inter-temporal spillovers, $\tau_i - b_i$, distort competitive equilibrium sectoral innovation rates towards upstream industries. Remember that firms discount future stream of profits at a higher rate, $\rho + \tau_i - b_i$, than social planner, ρ, because future innovators may steal incumbent firms' products. Suppose that ρ is small, and firms optimize only future gross profit streams, π, rather than the net profit stream, $\pi - c(b)$. Then, $\frac{C'(\tau_i)(\tau_i - b_i)}{C'(\tau_j)(\tau_j - b_j)} \approx \frac{\pi_i}{\pi_j}$. In equilibrium: $\tau_i - b_i = z_i = N\tau_i$[40] Hence, $\frac{\tau_i}{\tau_j} \approx \left(\frac{\pi_i}{\pi_j}\right)^{1/\gamma}$ as opposed to $\frac{\tau_i}{\tau_j} \approx \left(\frac{\pi_i}{\pi_j}\right)^{1/(\gamma-1)}$[41]. If industry i is more downstream than industry j, then $\pi_i/\pi_j > 1$. Since $\left(\frac{\pi_i}{\pi_j}\right)^{1/\gamma} < \left(\frac{\pi_i}{\pi_j}\right)^{1/(\gamma-1)}$, the inter-temporal spillover effect pushes sectoral innovations towards upstream industries.

Net–profit considerations. Third, the fact that firms care about net profits, $\pi - c(b)$ instead of gross profits, π, pushes equilibrium innovation rates towards upstream industries. Suppose that there is no inter-temporal spillover effect. Then, $\frac{C'(\tau_i)\rho + c(b_i)}{C'(\tau_j)\rho + c(b_j)} \approx \frac{\pi_i}{\pi_j}$. In equilibrium, $c(b) = M\tau^{\gamma}$.[42] Therefore, $\frac{A\tau_i^{\gamma-1}(\rho + \tau_i M/A)}{A\tau_j^{\gamma-1}(\rho + \tau_j M/A)} = \frac{\pi_i}{\pi_j}$. For this equality to hold, τ_i/τ_j should be small, relative to the case where firms consider only gross profit in their R&D decisions.

In summary, the inter-temporal spillover and net profit considerations of firms tilts innovation rates towards upstream sectors, while the monopoly distortion tilts innovation rates toward downstream sectors.

Figure 14 depicts a numerical example of the constrained social planner allocation.[43] The left panel shows that the social planner allocates more innovative resources to the most downstream industry, while reducing resources to other sectors. Note that the social planner reduces innovation rates in industries located in the middle of the supply chain at a higher rate than that of more downstream industries. However, the more upstream industries see a slightly lower reduction in innovation rates than that of midstream industries. Overall, this reallocation generates a 0.47% welfare gain by increasing the long-run growth rate of the economy from 1.56% to 1.58%.

The right panel of Figure 14 shows the impact of various externalities on the allocation of innovation across industries. Each line shows the innovation rates in a given case relative to the innovation rates in the constrained optimum allocation. Introducing the monopoly distortion (as in the red line in the right panel of the figure) increases the innovation rate in the most downstream industry but reduces innovation rates in the other industries, with more upstream industries facing larger declines.

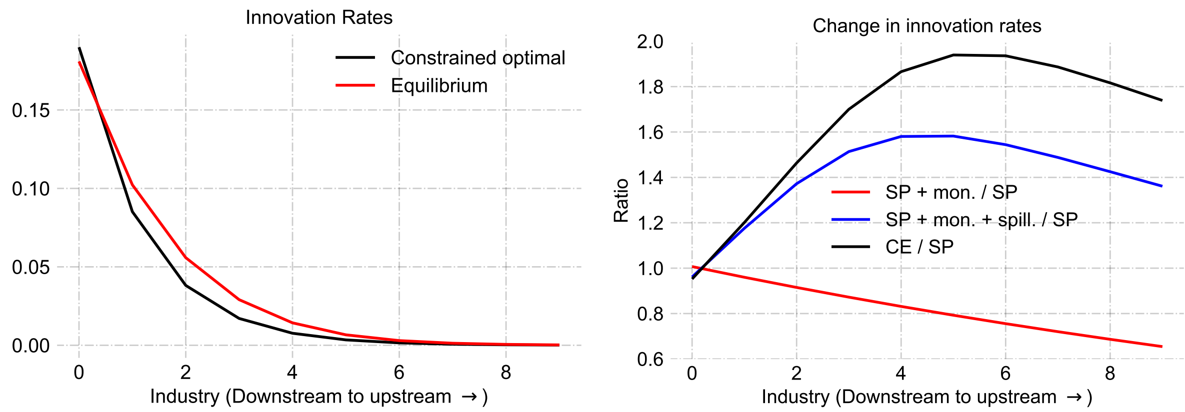
[40] $N \equiv \frac{\left(\frac{(1-s^i)\gamma}{(1-s^e)\psi}\right)^{1/(\gamma-1)}}{1+\left(\frac{(1-s^i)\gamma}{(1-s^e)\psi}\right)^{1/(\gamma-1)}}$.

[41] In the absence of inter-temporal spillover effect $\frac{C'(\tau_i)}{C'(\tau_j)} \approx \frac{\pi_i}{\pi_j}$, therefore $\frac{\tau_i}{\tau_j} \approx \left(\frac{\pi_i}{\pi_j}\right)^{1/(\gamma-1)}$.

[42] $M \equiv \chi_i \left(\frac{1}{\left(\frac{(1-s_i)\gamma}{(1-s_e)\psi_i}\right)^{1/(\gamma-1)}+1}\right)^{\gamma}$

[43] In this parameterization, there are $n = 10$ industries, $\alpha = 0$, $\sigma_i = .7$ for all but the most upstream industry, and $\sigma_n = 1$. Innovation functions are identical across sectors, with $\lambda_i = 1.074$, $\chi_i = 5.84$, $\psi_i = 2.29$, $\gamma = 2.5$.

Figure 14: Constrained Optimum Innovation Rates



Notes: Industry innovation rates in the competitive equilibrium and the constrained optimum social planner allocation are shown in the left panel. The right panel shows the effects of various distortions on the innovation rates. "CE/SP" refers to the competitive equilibrium innovation rate, relative to the constrained–optimum social planner innovation rate. "SP+mon./SP" refers to innovation rates when the social planner takes the monopolistic competition distortion as given, relative to the SP innovation rates. "SP+mon.+spill/SP" refers to innovation rates when the social planenr takes the monopolistic competition and inter-temporal spillover distortions as given, relative to the SP innovation rates.

This is because monopoly distortions accumulate on upstream industries.

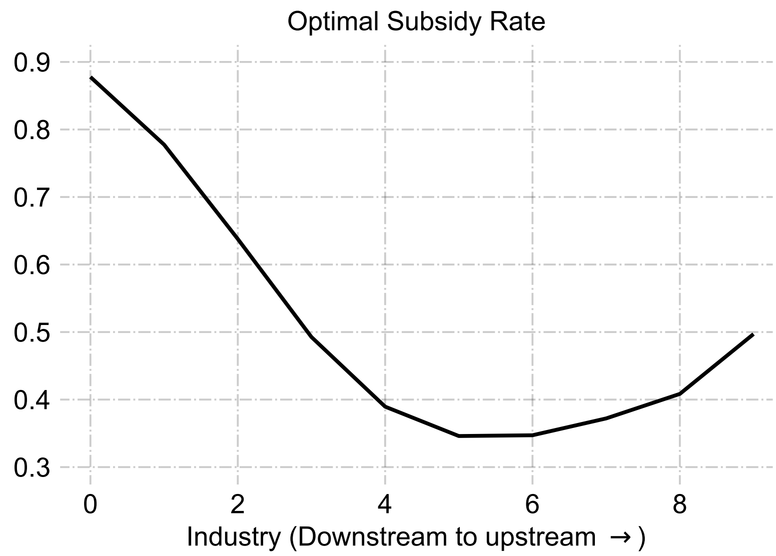
When inter-temporal spillovers are incorporated along with the monopoly distortions (as in the blue line in the figure), industry innovation rates of upstream industries increase, whereas innovation rate of the most downstream industry decrease. Further, the increase in industry innovation rates is stronger for downstream industries. Together, the monopoly distortion and inter-temporal spillover effect result in a non-monotonic increase in innovation rates relative to the social planner allocation. Innovation rates of industries in the middle of the supply-chain increase more than the innovation rates among upstream industries.

Lastly, introducing the net profit consideration yields the competitive equilibrium (as in the black in the right panel of the figure). The net profit consideration disproportionately increases innovation rates in upstream industries. These findings match the previous theoretical discussion of the distributional impacts of externalities: monopoly power reduces innovation in the upstream industries (relative to the social planner allocation), whereas inter-temporal spillover and net profit consideration increase innovation rates in the upstream industries.

H.2 Socially Optimal Innovation Subsidies

This subsection analyzes the optimal innovation subsidy to industries in the simple, ten-industry economy. Figure 15 shows the subsidy rates that maximizes the welfare gain. The subsidy rate schedule exhibits a U-shape, with the lowest subsidy directed at approximately the mid-point of the vertical supply chain.

Figure 15: Optimal Innovation Subsidy



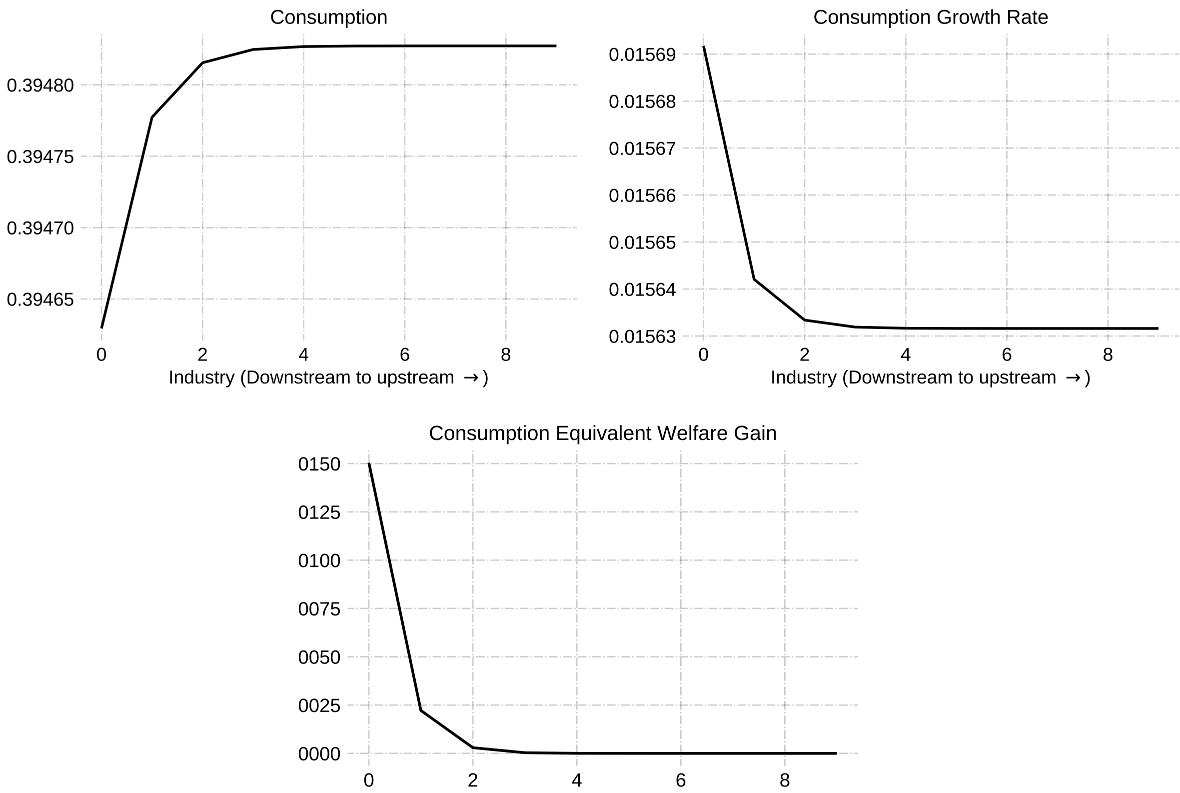
Notes: Optimal R&D subsidy rates to industries along the vertical supply chain.

To highlight the driving force behind this result, consider the impact of an extra 10% percent subsidy to one industry at a time. An increase in industry R&D subsidy lowers consumption in the initial period, while increasing the consumption growth rate. The elasticity of consumption growth rate with respect to the innovation subsidy in the industry in question decreases with the distance to final consumption. However, the magnitude of the elasticity of short–run consumption with respect to industry innovation subsidy also decreases with the distance to final consumption. Subsidizing downstream industries leads to a larger increase in the consumption growth rate, but also a larger decrease in the short run consumption. The former effect dominates, resulting in larger welfare gains. Subsidizing the most upstream industries does not increase the consumption growth rate as much, nor does it significantly decrease the level of short–run consumption. The trade–off between long–run growth and short–run reductions in initial consumption results in the U-shaped optimal innovation subsidy profile depicted in Figure 15.

I Growth Decomposition

Long–run consumption growth follows from innovation in the two sectors. As described in equation (23), the consumption growth rate can be decomposed into the contributions of technological progress in consumption and investment goods. By definition, the total innovation rate is the sum of entrant and incumbent innovation, so that the consumption growth rate can be further decomposed into the contributions of entrants and incumbents. Contributions to the growth rate can therefore be

Figure 16: 10% increase in industry innovation subsidy



Notes: The graphs show the effects of a 10 percent increase in the R&D subsidy rate in a given industry on key variables. Each point in the graphs is a separate exercise in which only the subsidy rate of the industry along the x-axis goes up, keeping subsidy rates in other industries fixed. The upper left panel shows the effect on consumption in the initial period; the upper right panel shows the effect on the growth rate of consumption at the balanced growth path; and the lower panel shows the corresponding welfare gain of the specified exercise.

decomposed by sector and incumbency

$$g_C = (z_c + b_c)\ln\lambda_c + \frac{\alpha}{1-\alpha}(z_x + b_x)\ln\lambda_x$$
$$g_C = \underbrace{z_c \ln\lambda_c}_{\substack{\text{Consumption} \\ \text{Entrants}}} + \underbrace{b_c \ln\lambda_c}_{\substack{\text{Consumption} \\ \text{Incumbents}}} + \underbrace{\frac{\alpha}{1-\alpha} z_x \ln\lambda_x}_{\substack{\text{Investment} \\ \text{Entrants}}} + \underbrace{\frac{\alpha}{1-\alpha} b_x \ln\lambda_x}_{\substack{\text{Investment} \\ \text{Incumbents}}}, \tag{51}$$

and are displayed in Table 17 as a percentage of the consumption growth rate. The investment sector contributes 77% percent of growth, whereas the consumption sector contributes 23%. The contribution of the investment sector in my estimates is higher than the estimates of Sakellaris and Wilson (2004), who empirically find that embodied technological change in investment goods accounts for two thirds of macroeconomic growth. Krusell (1998) develops an endogenous growth model that can account for the decline in the relative price of investment goods. He attributes approximately half of the consumption growth to investment specific technological change.

Table 17: Consumption Growth Decomposition

	Consumption	Investment	Total
Entrant	8%	25%	33%
Incumbent	15%	53%	67%
Total	23%	77%	

Entrants contribute approximately one third to growth. Recall that, in the model, entrants and incumbents are equally innovative. Hence, the difference in entrant and incumbent contribution to growth stems mainly from differences in entry and expansion rates. Foster et al. (2001) find similar results using the Census of Manufacturers data from 1977 to 1987. In particular, net entry contributes one quarter of multi–factor productivity growth[44]. Overall, investment sector incumbents contribute the most to growth and consumption sector entrants contribute the least. Intuitively, most of the growth comes from companies producing better machines, and less comes from consumption sector entrants like new restaurants.

[44]Notice that I do growth decomposition in this anaylysis, whereas Foster et al. (2001) analyzis contributions to productivity.