Worker Skills, Firm Dynamics, and Productivity

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Abstract

We develop a general equilibrium model of firm dynamics with multiple labour skills to analyse the productivity implications of skill shortages. Firms employ cognitive, interpersonal, and manual skills in both operational and overhead tasks, with sectoral variation in skill intensities, and within sector firm size heterogeneity due to idiosyncratic technology draws. Calibrated to UK data, the model reveals that cognitive skills are a binding constraint on productivity. A reduction in the supply of cognitive skills lowers labour productivity at all levels – firms, sectors, and in the aggregate. About half of the sectoral effects are due to changes in the within-sector distribution, whereby larger firms grow in relative size but become less productive. Sectoral heterogeneity in skill intensities amplifies these effects.

Keywords: multi-dimensional skills; skills shortages; firm dynamics; productivity

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1 Introduction

Skills shortages are a widely discussed labour market phenomenon with potentially far-reaching consequences for productivity. For instance, after the recovery from the Covid-19 recession, labour shortages became apparent in the UK where the number of job vacancies reached record highs in many industries (House of Lords, Economic Affairs Committee, 2022). Difficulties to hire employees with the right set of skills might suppress firms' output and (labour) productivity[1], as managers might be forced to allocate tasks to those who are not well-suited to perform these tasks.[2] Taking a more macroeconomic point of view, we expect that skills shortages to not just affect individual firms in isolation, but that competition over the scarce inputs results in changes in the distribution of firms, potentially affecting industrial sectors differentially. To understand the macroeconomic impact of skills shortages, all these effects need to be taken into account.

In this paper, we develop a quantitative general equilibrium model of firm dynamics that incorporates multiple types of worker skills, each utilised with varying intensities across sectors. Our framework builds on the Hopenhayn (1992) model of firm heterogeneity and dynamics, extended to include multiple skill types, which are employed in both production (operations labour) and overhead tasks. We calibrate the model using UK data on the skill composition of employment, firm size/productivity distributions, and firm entry statistics. We then simulate skill supply shocks to examine their impact on labour productivity across firms, sectors, and at the aggregate level.

Our analysis is motivated by the observation that skills are multi-dimensional. Following Lise and Postel-Vinay (2020), we focus on three dimensions of skills — cognitive, interpersonal, and manual — and introduce these into a model of firm dynamics. We note that categorising skills by these three types is different to splitting workers

[1]Throughout this paper when we write productivity we mean labour productivity unless stated otherwise.

[2]Jane Gratton, the Head of People Policy at the British Chambers of Commerce, recently expressed this concern as 'Because [firms] got staff shortages at lower levels in the organisation, qualified staff and higher skilled staff are doing lower-level work, which is taking their eye off the productivity goal. And also, they cannot release staff for training.' (Has Britain Stopped Working? – the Bottom Line, BBC).

into low and high skilled by wages or by education. We see in the data for instance that amongst the highest paid occupations the interpersonal skill intensity is the highest, with the intensity of cognitive skills only in second place. As we discuss below, our results reveal that variations in the supply of these two skills have quantitatively different effects, implying that pooling interpersonal and cognitive skills together as a single 'high skill' would be misleading in an analysis of skills shortages. While we do not model workers' labour supply as a bundle of multi-dimensional skills, in our framework firms' labour demand is multi-dimensional in cognitive, manual, and interpersonal skills, and we evaluate how variations in the aggregate skills supplies impacts firms and productivity.

Our benchmark counterfactuals focus on changes in skill composition, where reductions in the supply of one skill is accompanied by an equal magnitude increase in the supply of the other skills. The objective is to identify which skills are relatively scarce in equilibrium and therefore most consequential for productivity. Composition-shock experiments are our benchmark scenario because they correspond to counterfactuals for assessing priorities in education and training policies, where greater investment in one skill dimension typically comes at the expense of others. We also report results for absolute shortage counterfactuals in which one skill supply declines while the others remain fixed, as well as counterfactuals that allow for correlated changes in skill supplies. The resulting productivity effects are quantitatively similar across these alternative exercises. In all cases, reductions in cognitive skills generate the largest productivity losses, identifying cognitive skills as the quantitatively most important skill margin for productivity.

In principle, the framework could accommodate any number of industrial sectors, but for the quantitative model we concentrate on two broad sectors: production and professional services (including technology and media)[3]. Contrasting these two sectors is particularly interesting as they are markedly distinct; they differ in tasks, value added per worker, and in the characteristics of worker skills. Although both sectors have broadly similar shares of cognitive skills among their workers (with professional services having moderately higher shares), the production sector is far more manual-

[3]These two broad sectors account for more than half of the value added in the UK private sector.

intensive, while professional services rely much more heavily on interpersonal skills. The two sectors differ vastly in their firm size distribution too, with professional services having a much larger share of small employers than the production sector (see data in Figure 5).

Despite these differences in skill intensities, the sectors compete over the same pool of workers' skills. Through general equilibrium linkages, shocks to the aggregate skill composition impact firms in all sectors via changes in relative prices, firm entry, and employment composition, which in turn will shape the firm size distribution and impact overall productivity.

Our central finding is that cognitive skills currently represent a binding constraint on productivity in the UK economy, whereas interpersonal skills hardly do, despite being intensively used in high wage occupations. A reduction in the supply of cognitive skills, both as a composition or an absolute shock, lowers labour productivity at all levels – firms, sectors, and in the aggregate. While effects via firm exit are minimal, the main impacts arise from effects within and between active firms, including through firm entry. First, the average productivity of firms declines as their employment mix tilts away from the scarcer skill. Second, within sectors, there are differential effects across firm size: larger firms grow in relative size but also become less productive. This change across the distribution of active firms further depresses sectoral and aggregate productivity.

Most of the response to cognitive skill shortages stems from heterogeneity in how intensively the three distinct skills are used in operations.[4] In contrast, heterogeneity in how intensively the distinct skills are used in overhead labour has negligible explanatory power in the effects of skills shortages on productivity loss. Similarly, initial differences in the supply of each skill play a limited role in determining how subsequent skill supply shocks affect productivity.

Our paper connects several strands of economic literature. First, we build on the recent literature that emphasises the multi-dimensional nature of skills — describing individuals' abilities across a range of tasks. Weinberger (2014) and Dem-

[4]To avoid confusion between firm production and the production sector, we use the term 'operations' to refer to the former.

ing (2017) highlight the importance of interactions between social and cognitive skills for wages and productivity. Lindenlaub (2017) and Deming (2023) demonstrate how multi-dimensional skills shape worker sorting and the wage structure. We draw on this literature and, following Lise and Postel-Vinay (2020), focus on three types of skills: cognitive, manual, and interpersonal. However, our analysis differs: we study how the allocation of skills across heterogeneous firms affects productivity. Girsberger, Koomen, and Krapf (2022) also consider these three skill dimensions and analyse how workers' skill acquisition influences productivity and employment transitions in the context of heterogeneous workers. In contrast, we focus on the firm side of the market, examining how aggregate skill supplies shape firm dynamics and productivity outcomes.

Second, our paper contributes to the literature on firm heterogeneity. Hopenhayn (1992) and Hopenhayn and Rogerson (1993) provide the canonical model of firm entry, exit, and dynamics under idiosyncratic productivity shocks. We extend this framework by moving beyond a homogeneous labour input to examine the implications of skill heterogeneity. Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) demonstrate that the allocation of inputs across firms can have substantial effects on aggregate productivity. While our model differs in structure, we pursue a related idea by analysing how the allocation of scarce skills across heterogeneous firms shapes labour productivity. Mukoyama, Takayama, and Tanaka (2025) develop a model that incorporates multiple tasks (occupations) into the Hopenhayn (1992) and Hopenhayn and Rogerson (1993) framework. Although the model setups are similar, our focus differs: they examine the effects of firing costs on worker reallocation across occupations (where workers are ex ante homogeneous but moving workers across occupations is costly), whereas we study skills shortages in an environment with heterogeneous workers (but no firing costs). Gottlieb, Poschke, and Tueting (2025) study the role of skill endowments in explaining cross-country differences in firm dynamics. Similar to our work, they incorporate multiple skill types into the Hopenhayn (1992) and Hopenhayn and Rogerson (1993) framework. However, while their skill types are defined by differences in educational attainment, ours are based on workers' underlying abilities. Moreover, whereas they introduce a productivity-correlated returns-to-scale parameter and goods market distortions, we abstract from these features and instead focus on cross-industry variation in the effects of skill shocks by modelling distinct industries with sector-specific parameters.

Our paper is also related to a strand of the literature that studies determinants of the firm distribution. Hopenhayn, Neira, and Singhania (2022) analyse how demographic forces shape the firm distribution through firm entry, while Mandelman, Mehra, and Shen (2025) study how immigration-policy frictions affect the entry and survival of young firms. Our focus is different. Rather than emphasizing policy distortions or entry frictions, we develop a framework in which heterogeneous firms compete for multiple skills in competitive labour markets. In our model, aggregate changes in skill supplies affect equilibrium wages and firms' input choices. Due to overhead labour requirements, these changes affect firms differentially, altering the allocation of employment across firms. As a result, skill supply shocks reshape not only labour productivity within firms but also the allocation of employment across heterogeneous firms, and to a small extent firm exit.

Last, we relate to the literature on skills mismatch. Şahin, Song, Topa, and Violante (2014) quantify the impact of mismatch on unemployment, while Guvenen, Kuruscu, Tanaka, and Wiczer (2020) and Baley, Figueiredo, and Ulbricht (2022) examine how multi-dimensional mismatch affects productivity, abstracting from within-firm complementarities. In contrast, there is not mismatch in our model. We take a different approach: we specify sector-specific production functions in which firms use multiple skill types, allowing for substitution between them. This enables us to analyse how firm and sectoral dynamics respond to aggregate skills supply shocks, capturing within-firm adjustments in skill use.

In the next section we briefly present some stylized facts that suggests that skills shortages are indeed multi-dimensional and vary by industry. We then develop in section 3 our quantitative general equilibrium model of heterogeneous firms with multiple skills. After calibrating the model against detailed UK data in section 4, we evaluate in section 5 the role of cognitive, manual, and interpersonal skills for labour productivity. The final section concludes.

2 Motivating Facts

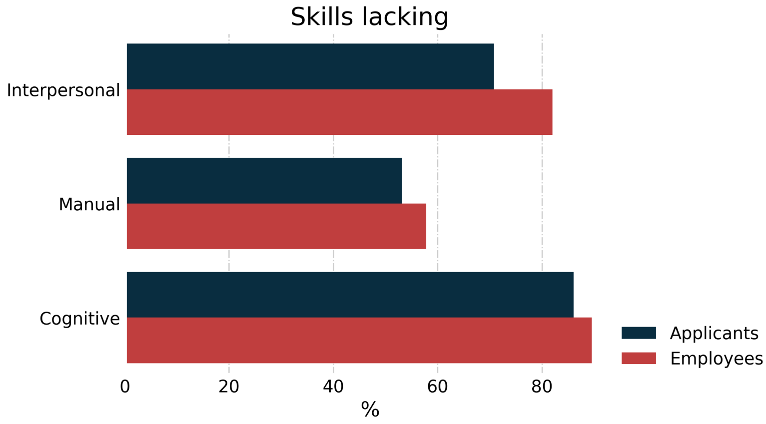
To motivate our analysis of skills shortages, we draw on data from the ONS Employer Skills Survey for the year 2022. This survey, conducted by the Office for National Statistics (ONS), collects establishment-level information from UK employers on a range of issues related to workforce skills, including whether current employees or job applicants are perceived to lack specific types of skills.

We classify the reported skills into three categories: cognitive, interpersonal, and manual skills. As discussed in the introduction, we focus on these three dimensions following Lise and Postel-Vinay (2020) and other recent work such as Girsberger et al. (2022). To provide suggestive evidence on skills shortages, we group the detailed skill items from the Employer Skills Survey into these categories through a manual mapping based on the ONS skill descriptions.[5] The detailed mapping is provided in Appendix F.

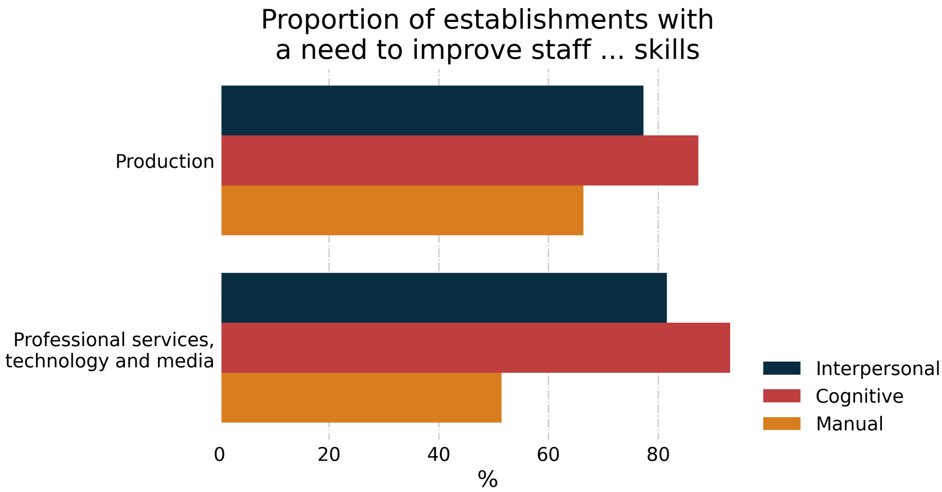
Figure 1a presents the prevalence of reported skill gaps among current employees and job applicants, by skill category. While shortages are reported across all skill types, lacking cognitive skills are the most prevalent, followed by interpersonal skills. This pattern suggests that a simple one-dimensional treatment of labour inputs is obscuring an analysis of skills.

Figure 1b breaks down reports of a need to improve workers' skills across the three dimensions by sector, distinguishing between employers in the production sector and those in professional services (including technology and media). Cognitive and interpersonal skill shortages are reported at broadly similar rates across both sectors. However, the need to boost workers' manual skills is substantially more pronounced in the production sector. The cross-sector variation in the survey responses suggests that skill intensities might differ. When we turn to the Annual Population Survey to calibrate our quantitative model in Section 4, we indeed find that the production sector is more manual skill-intensive, which reinforces the importance of accounting for both the multidimensional nature of skills and sectoral heterogeneity in any analysis

[5] For the calibration of our model, we construct skill shares by combining data from the Annual Population Survey on occupational employment shares (by industry) with the occupational skill requirements reported in Lise and Postel-Vinay (2020); see Section 4 for details.



(a) By Skill Category



(b) By Sector (for Current Employees)

Figure 1: Reported Skill Shortages in the UK

Source: ONS Employer Skills Survey 2022 and authors' calculations. The top figure: the survey asks about the skills found difficult to obtain from applicants. For example, "Soft / people skills found difficult to obtain from applicants (SUMMARY): Instructing, teaching or training people" (as quoted from the ONS Employer Skills Survey 2022). The survey also about employee skills which need improving. For example, "Soft / people skills that need improving (SUMMARY): Customer handling skills." We first group skills (the specific skills which come after ':') into three major groups: interpersonal skills, manual skills, and cognitive skills (the grouping is in the Appendix). Then we calculate, among the establishments which report having a skills shortage vacancy (a different question), the proportion of establishment which also report having found at least one of the skills in a skill group difficult to obtain (the bars labelled as 'Applicants'). We also calculate, among the establishment which report at least one instance of lack of proficiency among staff (a different question), the proportion of establishments which report having found at least of the skills in a skill group that needs improving (the bars labelled 'Employees'). The results are reported for the entire economy. The bottom figure reports the results by sector (the industry grouping we used in our calibration).

of skill shortages.

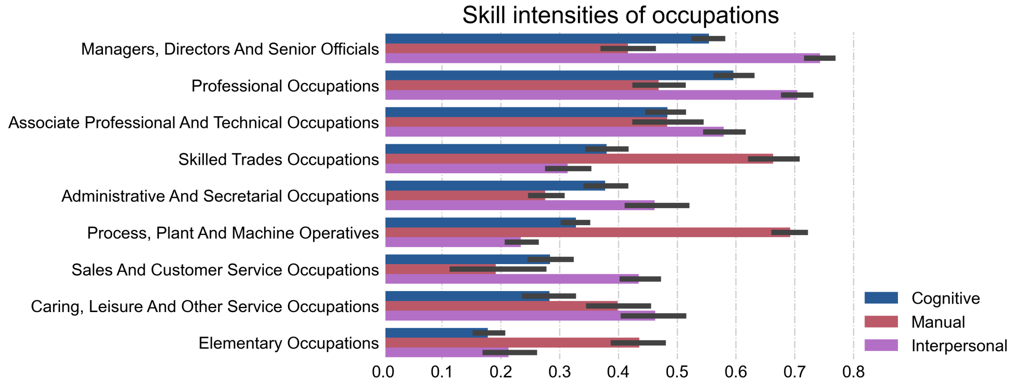


Figure 2: Skill Use by Occupation

Source: Lise and Postel-Vinay (2020) and authors' calculations. The figure shows the means and 95% confidence intervals (black bars) of skill intensities of major occupations. The figure is constructed by taking averages of 4-digit occupation skill intensities from Lise and Postel-Vinay (2020) at the major occupation group level.

Another way of gauging the importance of these three skills is to look at their prevalence in different occupations. Figure 2 shows the intensity of each skill by 1-digit occupation groups, ranked by average wages (in descending order)[6]. This graph is constructed by taking averages of 4-digit occupation skill intensities from Lise and Postel-Vinay (2020) at the major occupation group level. While the use of cognitive skills increases broadly with the rank of occupations, it is interpersonal skills that are the most important skill in the highest paid occupation (managers, directors and senior officials). At the same time, the use of interpersonal (and of manual) skills is not monotonic in occupation rank. These patterns highlight the nuanced nature of skills and the need to move beyond workers based on educational attainment alone.

3 Model of Firm Dynamics with Multiple Skills

To study the productivity implications of variations in the aggregate supplies of skills, we develop a general equilibrium model with firm dynamics. We introduce into the Hopenhayn (1992) and Hopenhayn and Rogerson (1993) model the novel feature that labour is differentiated by skill types. Following Lise and Postel-Vinay (2020) we con-

[6]Occupations are ranked in descending order of 2023 mean full-time gross hourly wages from the UK Annual Survey of Hours and Earnings (ASHE).

sider cognitive, manual, and interpersonal skills. We further add a two sector structure: production and professional services (including technology and media). Within each sector, firms are heterogeneous due to (Hicks-neutral) technology differences and operate under decreasing returns to scale to produce a homogenous sector-specific good. Sectoral output is consumed by a representative household with CES preferences across goods. The equilibrium is competitive, with free firm entry (subject to fixed costs) and exit, where all firms and consumers take prices and wages as a given.

The aggregate labour supply of each skill we take as exogenous because our interest lies in the endogenous assignment of skills to each (active) firm and in how this shapes productivity. Changes in the skill supply will alter wage rates for each of the skills, employment in each firm, entry and exit of firms, sectoral output prices and GDP, and thereby also labour productivity at all levels.

3.1 Model setup

We index sectors by j and firms by k. To ease notation, we omit the time index for now, and only introduces it as t in dynamic equations. Each labour skill is perfectly mobile across firms and sectors, such that there is a single wage rate w_i for each skill for $i \in \{C, M, I\}$ (cognitive, manual, and interpersonal skills, respectively).

Firms. Firms combine the various labour skills using a CES aggregator for production under decreasing returns to scale and in the presence of some fixed costs. We follow Hopenhayn (1992) in setting up the firm dynamic structure. Within a sector firms differ in idiosyncratic productivity in a Hicks-neutral technology way. Idiosyncratic productivity is drawn upon entry from a sector-specific distribution G_j and subsequently evolves according to an AR(1) process. Firms face fixed entry costs, denominated in units of the sector's output they seek to enter. They also face each period a fixed operating cost, which we set up as an overhead labour cost as a linear combination over the three skills. Firms make optimal decisions, including whether to exit at the end of the period (before the realization of next period's productivity). The mass of active firms is therefore endogenous.

Output of firm k in industry j is given by:

$$y_{kj} = s_{kj} L_{kj}^{\theta}, \quad \text{where } \theta < 1, \tag{1}$$

$$L_{kj} = \left(\sum_i \phi_{j,i}^{1/\sigma} n_{k,i}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}, \tag{2}$$

$i \in \{C, M, I\}$ denotes cognitive, manual, and interpersonal skill types, and σ is the elasticity of substitution across skills. The returns to scale parameter θ is below one such that firms have increasing marginal costs. Higher values of the Hicks-neutral technology parameter s_{kj} result —at given inputs— in lower (marginal) production cost. As in equilibrium all firms in a sector face the same product price p_j, which implies that firms with higher s_{kj} are larger in terms of value-added and composite employment. Consequently within a sector the distribution of firm size is closely linked to the distribution of s_{kj} amongst the active firms, where both firm size and idiosyncratic productivity distributions are endogenous. While the firms' technology evolves according to an $AR(1)$ as

$$\log(s_{t+1}) = (1-\rho)\mu_j + \rho \log(s_t) + \varepsilon_{t+1}, \text{ with } \varepsilon \sim N(0, \sigma_j^2) \text{ and } 0 < \rho < 1, \tag{3}$$

firms decide optimally at the end of each period (after production took place, but before the realization of next period's draw) whether to stay or exit the market. As we will see below, because of overhead fixed costs this structure implies a threshold productivity, such that firms with $s_{kjt} < s^*_{jt}$ leave. It is always the least productive firms that leave, but what the critical value s^*_{jt} is determined in general equilibrium.

Our approach differs from Hopenhayn (1992) in two key ways. First, in our model L_k is not a one-dimensional labour input but a CES composite of the three distinct skill types: interpersonal, cognitive, and manual. Second, the fixed cost of operations stems from the overhead labour required. We model this as a (time-invariant) linear combination across the skills, parametrised as $C_{f,j} \sum_i n_{f,i} w_i$, where w_i the (time-varying) equilibrium wage for skill i and $\{n_{f,i}\}$ the required units of overhead labour. $C_{f,j}$ is a scaling parameter that determines the overall size of the overhead labour requirement in sector j.

The optimal choices by firms are all static apart from the exit decision. As we show in the appendix, firm k's optimal demand for each skill i used in production of output satisfies

$$n_{kij} = \phi_{j,i} \left(\frac{w_j}{w_i} \right)^{\sigma} L_{kj} \text{ for } i \in \{C, M, I\}, \tag{4}$$

where $w_j = \left(\sum_i \phi_{j,i} w_i^{1-\sigma} \right)^{1/(1-\sigma)}$ is the unit cost of composite labour L_{kj} in sector j, with the property that $\sum_i w_i n_{k,i} = w_j L_{jk}$. In the following we refer to these demands for labour skills, as inputs to the production of output, as a firm's *operations labour* demand (one for each skill). These $\{n_{kij}\}$ are variable amounts reflecting the optimization by firm k, in contrast to the fixed overhead labour ($\{n_{f,i}\}$) required for firms to be active. We see that the skill shares in the operations labour mix of a firm reflects relative wages for each skill (w_i) and skill intensities ($\phi_{j,i}$), where the latter vary across sectors (j).[7]

The optimal size of a firm, in terms of composite labour (2) and output (1), is given by

$$L_{kj} = \left(\frac{p_j s_{kj} \theta}{w_j} \right)^{\frac{1}{1-\theta}} \tag{5}$$

$$y_{kj} = s_{kj}^{\frac{1}{1-\theta}} \left(\frac{p_j \theta}{w_j} \right)^{\frac{\theta}{1-\theta}}. \tag{6}$$

Since within a sector only s_{kj} varies across firms and $0 < \theta < 1$, firms with higher technology draws have in equilibrium larger overall employment, larger employment in each skill, and larger value-added compared to other firms in their sector.

The dynamic part in the firm optimization problem stems from deciding whether to exit the market at the end of the period, before next period's productivity has been realized. This dynamic problem can be represented a value function. To ease the notation, we omit the firm index k from the choice variables and the sector index j from the value function, and use primes to denote next period values:

$$v(s, p_j, \{w_i\}) = \max_L p_j s L^{\theta} - C_{f,j} \sum_i n_{f,i} w_i - w_j L + \beta \max\{0, E_s v(s', p_j', \{w_i'\})\}, \tag{7}$$

[7] The composite wage index w_j is sector-specific, even though the wage rates for each skill are common across sectors, because the skill intensities $\{\phi_{j,i}\}_i$ differ by sector.

where w_j is the unit cost of labour in industry j, p_j the sectoral output price, w_i is the wage rate for skill type i, and $n_{f,i}$ is the share of skill type i in overhead labour. If a firm decides to exit the market, it receives a scrap value of 0. If it decides to stay, it gets the continuation value $E_s v(s', p'_j, \{w'_i\}) = E\left[v(s', p'_j, \{w'_i\})|s\right]$ where the expectation for the next period value over possible productivity realizations s', is conditional on the current value of technology s. Under the $AR(1)$ process for firm productivity (3), $E_s v(s', p'_j, \{w'_i\})$ is monotonically increasing in s, and due to the fixed overhead labour costs it is negative for small s and positive only for sufficiently high s. Firms with productivity $s < s^*$ exit the market where the critical threshold productivity is such that $E_{s^*} v(s', p'_j, \{w'_i\}) = 0$, while firms with $s \geq s^*$ stay.

We summarised the decisions of incumbent firms above. Left to describe are the entry dynamics. As typical in the Hopenhayn model, we assume that potential entrants pay an upfront entry cost and draw a productivity afterwards. The entrants are therefore not ex ante selected.[8] Specifically, we assume that potential entrants pay a cost of entry, $C_{e,j}$, in units of their industrial sector's output. Entering firms then draw their initial (log) productivity from a log-normal distribution G_j with mean $\mu_{e,j}$ and variance $\sigma^2_{e,j}$. Following the traditional Hopenhayn model, entrants, unlike incumbents, do not face a further fixed cost to operate in their first period in the market. Therefore, the value of entry is

$$v_e(p_j, \{w_i\}) = \int_s \left(v(s, p_j, \{w_i\}) + C_{f,j} \sum_i n_{f,i} w_i \right) dG(s). \tag{8}$$

Households. Workers belong to a representative household. The household supplies $\{N_i\}_{i \in C,M,I}$ units of labour inelastically, owns the firms and collects their profits (net of entry costs) and consumes output of each sector to maximise utility. We assume preferences over the sectoral goods are given by

$$U = \left(\sum_j \eta_j^{1/\epsilon} C_j^{(\epsilon-1)/\epsilon} \right)^{\epsilon/(\epsilon-1)}.$$

[8] Ex post, some firms that enter will make losses. Nonetheless, they will produce in their initial period, as this is optimal for minimising losses. If $E_s v(s') < 0$ they will leave the economy after one period, just like the other firms discussed above.

The household maximises utility by choosing consumption of each good C_j subject to the budget constraint

$$\sum_j p_j C_j \leq \sum_i w_i N_i + \Pi. \tag{9}$$

Optimal choices imply

$$\frac{C_i}{C_j} = \frac{\eta_i}{\eta_j} \left(\frac{p_j}{p_i} \right)^{\epsilon}. \tag{10}$$

3.2 Equilibrium

We solve for a stationary competitive equilibrium in which the distribution of firms and prices are constant over time. Firms and household optimise, and prices are such that all markets clear.

Let F_j denote the stationary distribution of incumbent firms over productivity draws s in sector j and M_j^f be the total mass of firms in industry j. G_j is the probability distribution over idiosyncratic productivity s from which new entrants draw. The equilibrium is such that:

- Given prices $\{p_j\}$ and wage rates $\{w_i\}$ potential entrants make optimal entry decisions in each sector, resulting in a measure M_j^e of entrants to sector j, such that free entry equates the value of entry to the cost of entry

  $$v_{e,j}(p_j, \{w_i\}) = p_j C_{e,j}$$

  where the entry cost $C_{e,j}$ is in units of output of the sector chosen by the entrants.

- Given prices $\{p_j\}$ and wage rates $\{w_i\}$ firms choose labour demand for each skilled labour and decide on whether to stay in the market.

- Given prices $\{p_j\}$ and wage rates $\{w_i\}$ households make consumption decisions.

- The market of goods clears for each sector j:

  $$M_j^f \int_s y_j(s) dF_j(s) + M_j^e \int_s y_j(s) dG_j(s) - C_{e,j} M_j^e = C_j,$$

where M_j^e is the measure of entrants, each of whom pays the sectoral output entry cost $C_{e,j}$.

- The labour market for each skill i clears

$$\sum_j \left[M_j^f \int_s n_{j,i}(s) dF_j(s) + M_j^f C_{f,j} n_{f,i} + M_j^e \int_s n_{j,i}(s) dG_j(s) \right] = N_i \text{ for } i \in \{C, M, I\},$$

where the total labour demand for a skill is the sum across all sectors and comprised of the operations labour demand by incumbent firms (first integral), the overhead labour by incumbents, and the operations labour demand by new entrants (last term).

4 Calibration

For the quantitative analysis we calibrate our general equilibrium model to match various aspects of the UK data over 2009–2023. Our strategy for calibrating the firm dynamics part is similar to Hopenhayn et al. (2022); additionally we incorporate the three labour skills. We externally calibrate some parameters and choose the remaining ones to match in the stationary equilibrium key aspects of the UK economy. Given our research angle, we specifically target statistics on skill intensities, firm size distributions in terms of employees, firm entry, productivity and survival, and sectoral value-added shares.

We use multiple datasets from the Office for National Statistics and UK Government to collect relevant targets for the UK economy. We target averages of available data over the years 2009 to 2023, but due to data availability the sample period used for each target differs (see Table A2 in Appendix A). The list of datasets and the information collected is

- Business Population Estimates for the United Kingdom (BPE) — mean employment of sectors and size distribution of firms,
- Business demography, quarterly, UK Statistical Bulletins — mean entrant employment.

- Business demography, annual UK, — entry rate, entry rate of employer firms, 5-year survival rate of entering firms

- Annual Business Survey (ABS) — average labour productivity of industrial sectors, the relative productivity of 250+ employee firms

- UK National Accounts, The Blue Book — GVA shares

- Labour Force Survey (LFS) — occupation shares of industries

The first four sources give us information about firms by industry. Across datasets, we assign industries consistently to our two sectors: production and professional services (including technology and media). Due to differences in industry coding across datasets, we assign media and technology throughout to professional services, as this aggregation gives us a consistent definition of sectors across the datasets. Then we use the generated firm-level statistics by sectors as calibration targets. We proceed similarly with sectoral data on gross value-added (from the national accounts) and on labour productivity (from the ABS).

The last data source is the Labour Force Survey (LFS). From this household survey, we construct in a first step occupational employment shares by industries. We then multiply these by occupational requirements for cognitive, manual, and interpersonal skills from Lise and Postel-Vinay (2020), which gives us the skill shares in employment in each sector. We also use this approach to construct the aggregate labour supplies for each skill, which we construct by summing across all workers.

	Definition	**Value**	**Source**
β	Discount factor	0.96	Hopenhayn et al. (2022)
θ	Elasticity of output w.r.t. labour	0.64	Hopenhayn et al. (2022)
ρ	Persistence of AR(1) productivity	0.984	Hopenhayn et al. (2022)
σ	b/w skills substitutability	0.73	Adachi (2025)
ϵ	sectoral consumption substit.	0.2	Bárány and Siegel (2020)
$\{N_i\}$	labour supply by skill (C, M, I)	0.201, 0.201, 0.198	labour force, times skill shares
$\{n_{f,i}\}$	overhead skill shares (C, M, I)	0.32, 0.25, 0.43	Managers' skill shares, Lise and Postel-Vinay (2020)

Table 1: Externally Calibrated Parameters

We start the model parametrisation by fixing some parameters based on the literature. From Hopenhayn et al. (2022) we take the elasticity of output with respect to overall labour as 0.64, and the persistence of Hicks-neutral technology in the $AR(1)$ process as 0.984. The elasticity of substitution across the sectoral consumption goods we take from Bárány and Siegel (2020) as 0.2, but we point out that this value has virtually no effect on our results (see robustness checks in section 5.3). More important is the production elasticity of substitution between skills. While papers focusing at aggregate level substitution between worker skill types have found values above one, at the firm level the evidence points to values less than one, which implies that the different skills are complements. We follow Adachi (2025) and set the elasticity between cognitive, manual, and inter-personal skills to 0.73.

	Definition	**Target**
$\{\phi_{j,i}\}$	Skill intensities	Skill shares
$C_{e,j}$	Cost of entry	Entry rate
$C_{f,j}$	Scale of overhead labour requirements	Rel. productivity of 250+ employee firms
μ_j	Mean of productivity $AR(1)$ process	Relative productivity of industries
σ_j^2	Variance of productivity $AR(1)$ process shocks	5-year survival rate
$\mu_{e,j}$	Mean of entrant productivity distribution	Mean employment of entrants
$\sigma_{e,j}^2$	Variance of entrant productivity distribution	Mean employment of sectors
η_j	Industry intensity in consumption	Gross value added shares

(a) Parameters to be Calibrated (by sector)

	Production	**Prof serv, tech & media**
ϕ_c	0.55	0.57
ϕ_m	0.20	0.12
ϕ_i	0.25	0.30
C_f	1.28	1.59
C_e	47.47	41.20
μ_i	-3.18	-3.93
σ_i	0.29	0.32
μ_e	-0.06	1.23
σ_e	0.95	0.55
η	0.31	0.69

(b) Calibrated Values

Table 2: Calibration Parameters

We construct the labour supplies of the various skills by multiplying the labour force participation rate of roughly 60 percent by the aggregate skill shares, which we compute by multiplying occupational shares from the LFS[9] with skill intensities from Lise and Postel-Vinay (2020). That is, we multiply 0.6 by the C, M, I shares of

[9]In this calculation only the production and professional service sectors are used to calculate occupitional employment shares.

$[0.335, 0.335, 0.33]$ to obtain the per capita labour supply by skill as $0.2011, 0.2011, 0.1978$ respectively. For the required overhead labour in each skill, we proceed by viewing the overhead requirement as the need to have managers. We assign the normalized skill intensities of managerial occupations from Lise and Postel-Vinay (2020) as the overhead skill shares $n_{f,i}$. Table 1 lists all externally calibrated parameters.

We are then left with 10 parameters for each industrial sector, shown in Table 2a, which we calibrated in general equilibrium against an equal number of targets in the data. We choose these to minimize the distance between the model-implied values and the data targets of Table 3. The table further shows the model statistics generated by the resulting calibrated parameters (whose values we report in Table 2b).

As the model is calibrated jointly, there is generally no one-to-one mapping between individual parameters and particular targets, but the calibration jointly chooses parameters to minimize the distance between the model and the data across all targeted moments. Consider, for instance, the skill-intensity parameters $\{\phi_{j,i}\}$. While these parameters directly affect firms' demand for different skills in operations labour, observed employment shares depend not only on skill intensities but also on equilibrium skill prices. These prices are jointly determined by the exogenous skill supplies and firms' endogenous labour demands, which in turn depend on the equilibrium firm distribution. As a result, employment shares reflect the interaction of skill intensities and equilibrium wages rather than the production-function parameters alone. Consequently, similar employment shares across skills do not imply similar values of the underlying productivity weights. A skill can have a relatively large production intensity but have only a moderate share in employment if that skill commands a relatively high price in equilibrium. This is precisely what occurs in the calibration: while cognitive skills account for roughly one-third of employment, they are the most expensive skill in equilibrium. Therefore, to match the observed employment shares, the calibration puts a relatively high intensity on cognitive skills in production.

Our calibrated model replicates the targeted moments very well. The only exception is a slight under-prediction of relative productivity for large firms in the production sector. The model does also reasonably well in replicating some untargeted moments: it generates the general patterns of firms' business shares by employment size bands. We show these (alongside results from experiments) in Figure 5 below as "Benchmark" model outcomes. Comparing these against the data, we see that the calibrated model qualitatively perfectly mimics the distribution of business shares across employment bands in each sector. Quantitatively, the model and data shares are very close in professional services, while in the production sector the model somewhat understates the share in the 1–4 employee band, but nonetheless gets the relative sizes broadly right.

	Data		**Model**	
	Production	Prof serv, tech & media	Production	Prof serv, tech & media
Mean employment	20.73	11.16	20.87	11.15
Mean entrant employment	4.08	2.51	4.06	2.51
Entry rate (employer)	0.10	0.12	0.10	0.12
5-year survival rate	0.39	0.44	0.40	0.44
GVA share	0.41	0.59	0.41	0.59
Rel. industry productivity	0.98	1.03	0.97	1.02
Rel. productivity of 250+	1.20	1.19	1.06	1.14
Cognitive share	0.31	0.35	0.31	0.35
Manual share	0.40	0.27	0.41	0.28
Interpersonal share	0.28	0.37	0.28	0.37

Table 3: Data and Model Moments in Calibration

In Table 3 we can also see that the two sectors are indeed different in terms of the data we are targeting. Compared to production, the average firm in professional services (including technology and media) has lower employment and relies more on cognitive and interpersonal skills. In the professional services sector, there is more frequent entry of firms and their average survival rates are higher. Compared to production, professional services account for a larger share of gross value-added (GVA) in the economy, and the sectors' labour productivity is slightly higher.[10] The sectoral differences in the data we target are reflected in the parameters obtained through the model calibration. Table 2b shows that the two sectors differ in various dimensions: (i) in the production skill intensities $\{\phi_{j,i}\}$ (reflecting skill shares), (ii) overhead labour fixed costs $C_{f,j}$ (higher in professional services where the typical firm is smaller), (iii) entry costs $C_{e,j}$ (lower in professional services where entry is more frequent), (iv) the parameters of the productivity process for incumbents and (v) of entrants (all feed on

[10] Note that the construction of these sectoral shares and relative industry productivity for the calibration is based on an economy from which all other sectors were excluded.

to relative productivity within and across sectors, survival rates, and employment distribution within a sector), as well as (vi) the consumption intensities (matching sectoral value-added shares).

The calibrated model also performs well with respect to non-targeted moments related to the remuneration of different skills. As an additional validation exercise, we compare the model-implied skill price ranking to wage premia observed in the UK data. Using occupational mean wages from the Labour Force Survey (2011–2020) and the same occupation–skill mapping employed throughout the calibration, we estimate reduced-form wage regressions relating occupational wages to cognitive, interpersonal, and manual skill intensities. The estimated wage premia imply interpersonal-to-cognitive and manual-to-cognitive wage ratios of 0.394 and 0.190, respectively. These are remarkably close to the corresponding model-implied values of 0.415 and 0.171. Details are reported in Appendix B. Since these wage moments are not directly targeted in the calibration, this provides additional support for the quantitative model's fit.

That the model generates equilibrium skill prices that closely resemble those observed in the aggregate UK economy also supports the use of our two-sector framework. Although the model abstracts from other sectors, firms are affected by aggregate skill shortages through equilibrium skill prices. The fact that the model reproduces the observed relative wage premia therefore suggests that it captures the economy-wide scarcity forces relevant for analysing how firms in the production and professional-services sectors respond to changes in aggregate skill supplies.

By giving a realistic firm size distributions, matching cross sectional facts, and replicating the allocation of skills to industries well, the model provides a suitable laboratory to simulate productivity effects induced by aggregate skill supply shifts. Given the sectoral differences we established here, we can expect to see differential responses across the sectors.

5 Quantitative Results

5.1 The Effects of Variations in the Aggregate Skills Supplies

In this section, we use our quantitative model to analyse the effects of changes in the aggregate supply of skills on (labour) productivity, which we calculate as the real output over the number of employees (including both operations and overhead labour), whether at the firm or sector level. Productivity effects can occur due to adjustments in various margins. When a skill becomes scarcer, the higher wage rate for this skill alters the skills mix at the firm level and at the same time firms' production costs. As a consequence, some firms may decide to exit, others to enter the market, and the number of active firms changes. But there will also be reallocations between incumbent firms and between the industrial sectors. These effects are non-trivial because in our model calibration industrial sectors differ in skills intensities, fixed costs, and underlying productivity processes. All these endogenous adjustments feed onto the measure of and distribution of active firms, sectoral productivity, and ultimately aggregate labour productivity or GDP per worker.

Exercise	Cognitive	Manual	Interpersonal
Benchmark	0.2011	0.2011	0.1978
Composition shocks			
-10% shock to cognitive	0.1810	0.2111	0.2079
-10% shock to manual	0.2112	0.1810	0.2079
-10% shock to interpersonal	0.2110	0.2110	0.1781
Absolute shocks			
-10% shock to cognitive	0.1810	0.2011	0.1978
-10% shock to manual	0.2011	0.1810	0.1978
-10% shock to interpersonal	0.2011	0.2011	0.1781

Table 4: Labour Supply Scenarios

We exogenously reduce the inelastic supply of each skill, one at a time, to examine the productivity consequences of skills shortages. This table shows the labour supply in our benchmark economy, under skill composition shocks and under absolute socks (one for each skill).

Specifically, we conduct a series of counterfactual exercises in which the supply of one skill is reduced by 10%, one skill at a time. In the first set of exercises, we consider *skill composition shocks*: a reduction in one skill is accompanied by an equal increase in the supply of the other two skills, such that total labour supply remains unchanged and only the composition of skills varies.[11] In the second set of exercises, we consider *absolute skill-supply shocks*, in which the supply of one skill is reduced while the supplies of the remaining skills are held at their benchmark levels. Table 4 reports the exact values used in each scenario.[12]

Our primary interest lies in the skill composition shocks, which allow us to identify which skills are relatively scarce in equilibrium and therefore most important for productivity. Such changes in relative skill supplies could arise, for example, from shifts in education curricula or training programmes that place greater emphasis on developing one particular skill at the expense of others. More broadly, changes in educational priorities, migration patterns, demographic change, or labour-force participation can all alter the supply of skills in the economy. The absolute shock exercises provide therefore a useful complement. The key difference between the two scenarios is whether a shock to one skill also affects the relative supplies of the other skills: composition shocks alter the entire skill mix, whereas absolute shocks leave the supplies of the remaining skills unchanged.

Such skills supply shocks naturally alter relative wages across the different skill types in order to restore equilibrium in labour markets. As firms' labour demand and production costs are impacted, the distribution of the mass of active firms changes, both due to reallocations between incumbents and due to effects on firm entry and exit. These changes occur in all sectors. But due to the various layers of structural sectoral differences, which we discussed in the model calibration in section 4, not to the same extent. This implies differential changes in sectoral output prices as well. We report all the effects on equilibrium wages and prices in Appendix Table A4. Here, we focus on the effects on labour productivity.

[11]Suppose the supply of cognitive labour is reduced by 10%, corresponding to an x percentage-point decline in the cognitive share of total labour supply. To keep aggregate labour supply constant, we increase the supplies of manual and interpersonal labour by $x/2$ percentage points each.

[12]In Appendix E.3, we further report the results of a correlated shock in which cognitive skills are reduced alongside a smaller reduction in interpersonal skills, with the latter calibrated using the empirically observed correlation between the two skills.

Sectoral level. We start by reporting the effects on sectoral real labour productivity in Table 5. Real labour productivity is measured as sectoral value added divided by total sectoral employment.[13] To facilitate comparison across counterfactuals, the table reports productivity as a relative productivity index, normalized to one in the benchmark economy. Additionally, the last column reports an index for aggregate real labour productivity, which we construct by adding the sectors' value-added under the initial benchmark economy's prices and dividing by the sum of sectors' employments.[14]

Exercise	Sectors: Production	Sectors: Prof. Services	Aggregate
Benchmark	1.000	1.000	1.000
Composition shocks			
-10% shock to cognitive	0.947	0.955	0.951
-10% shock to interpersonal	1.003	1.009	1.006
-10% shock to manual	1.041	1.026	1.033
Absolute shocks			
-10% shock to cognitive	0.964	0.970	0.967
-10% shock to interpersonal	1.003	1.007	1.005
-10% shock to manual	1.029	1.018	1.023

Table 5: Sectoral and Aggregated Real Labour Productivity

Real productivity in each sector is calculated as real total output (of incumbent and entrant firms) over total employment (including both operations and overhead labour) in the sector. We calculate aggregate productivity as the sum of real value added of sectors (calculated at benchmark economy prices) over total employment. Reported values are relative to the benchmark.

The productivity effects depend critically on which skill becomes scarcer. Recall that the composition-shock scenarios reduce the supply of one skill while increasing the supplies of the other two skills by an equal amount such that the total labour supply remains constant. By contrast, the absolute-shock scenarios reduce the supply

[13] In terms of model objects, we compute real labour productivity in sector j as

$$\frac{M_j^f \int_s y_j(s)dF_j + M_j^e \int_s y_j(s)dG_j}{\sum_{i\in C,M,I}\left[M_j^f \int_s n_{j,i}(s)dF_j + M_j^f C_{f,j} n_{f,i} + M_j^e \int_s n_{j,i}(s)dG_j\right]}$$

[14] The table shows the results on each sectors' real labour productivity (measured in the sector's own units of output). For cross-sector comparisons, we also construct revenue productivities and compute their ratios to the aggregate economy (spanning the model's two sectors). Appendix Table A7 reports these results. The productivity changes in this table reflect changes in not only in real productivity but also in relative sectoral prices.

of a single skill while holding the supplies of the remaining skills fixed.

In both types of counterfactuals, a reduction in cognitive skills lowers labour productivity in the production sector, the professional-services sector, and the aggregate economy. Under the composition shock, a 10% decline in cognitive skills reduces productivity by approximately 5% in both sectors and in the aggregate. The corresponding absolute-shock exercise yields slightly smaller, but still substantial, productivity losses of around 3–3.5% in all parts of the economy.

By contrast, reductions in manual skills or in interpersonal skills increase productivity in both sectors and in the aggregate economy. A 10% reduction in manual skill supply raises productivity by between 1.8% and 4.1%, depending on the sector and counterfactual considered. Reductions in interpersonal skills have also positive but much smaller effects, generating productivity changes by less than 1%. Overall, the results are very similar across composition and absolute shocks. They consistently identify, among the three skill dimensions, cognitive skills as the quantitatively most important determinant of productivity in the calibrated economy and as the only one for which an decrease in supply lowers productivity.

These results directly establish that the skills composition of the economy matters. Our quantitative model suggests that currently, cognitive skills are in shortage, as if the average worker had a higher share of cognitive skills (a skill composition shock), productivity would increase in all sectors and in the aggregate. In contrast, manual skills are in abundance — reducing their relative supply would boost productivity. The counterfactual simulation of the manual supply shock also highlights that lowering the manual skill share would benefit the production sector more than the professional services.

The magnitudes of the effects differ across composition and absolute shocks, even though the ranking of skills remains unchanged. The reason is that a composition shock induces a larger change in relative skill scarcity than an absolute shock. Consider a reduction in cognitive skills. Under the composition-shock exercise, the supply of cognitive skills falls while the supplies of manual and interpersonal skills increase. Under the absolute-shock exercise, only cognitive-skill supply falls, while the supplies of the other skills remain unchanged. Consequently, the composition shock generates a larger increase in the price of cognitive skills (relative to sectoral output prices) and a larger decline in the relative prices of the remaining skills. This can be seen in Appendix Table A5, where the increase in cognitive real wages is larger under the composition shock than under the absolute shock in both sectors. Firms therefore substitute away from cognitive skills more strongly under the composition shock. Since cognitive skills are the most productive and relatively scarce skill in the calibrated economy, this stronger shift towards manual and interpersonal skills leads to a larger decline in productivity. As a result, reductions of cognitive skills have somewhat larger productivity effects under composition shocks than under absolute shocks.

Note, a change in aggregate labour supply that leaves relative skill supplies unchanged has no effect on labour productivity in our model. Relative skill prices, firms' employment choices, exit decisions, and the firm distribution remain unchanged. The adjustment occurs solely through the measure of active firms, which scales proportionally to equate aggregate labour demand with labour supply. As a result, sectoral and aggregate output and employment change by the same factor, leaving labour productivity unaffected. Differences between the composition and absolute-shock exercises therefore arise from changes in relative skill scarcity rather than from changes in aggregate labour supply.

In the following we disentangle the productivity effects within each sector to establish how they arise. The overall sectoral effect stems from within-firm changes, changing firm sizes (reallocations between incumbents), entry and exit. As the effects of skill composition and absolute skill supply shocks are so similar and qualitatively identical, both at the sector and the firm level, we focus in the remainder of the main text on composition shocks and report the effects of the absolute shocks in Appendix D.1.

Decomposing sectoral productivity. We first use an Olley and Pakes (1996) style decomposition of a sector's aggregate labour productivity into an (unweighted) average of productivity at the firm level and the covariance between the firms' employment share and their productivity.

$$Productivity_j = \overline{Productivity_j} + \sum_{k \in j} (\omega_{kj} - \overline{\omega_j}) \left(Productivity_{kj} - \overline{Productivity_j}\right) \quad (11)$$

where the weights ω_{kj} are the shares of firm k in total employment (across all skills) in sector j. $\overline{Productivity_j}$ denotes the mean productivity across all firms in the sector and $\overline{\omega_j}$ is the mean employment share.[15] Comparing across model simulations the simple mean, the first term in (11), captures what happens to mean productivity across firms (when all are given equal weight regardless of their size), whereas the covariance, the second term in the equation, changes when firm sizes change systematically with productivity across firms. This covariance term captures the assignment of employment to firms that vary in their labour productivity. It captures the impact of the joint distribution of firms' size and productivity to aggregated productivity, and it increases if the correlation between firms' employment shares and their labour productivity increases. An increase in the covariance therefore indicates a change in the allocation of labour to the more productive firms (all within a sector).[16]

Table 6 reports the decomposition results for the skill composition shocks. We see that for manual and cognitive skills composition shocks both components of the Olley–Pakes decomposition, mean productivity and the covariance term, are impacted qualitatively in the same way. The effects of cognitive and manual skills reductions on sectoral productivities, as found above, are therefore driven by both within-firm and between-firm changes, all of which go in the same direction. However, there are some quantitative differences. For instance, in the case of cognitive skill composition shocks, changes in the allocation account for 55 percent of the labour productivity effects in the production sector, but only about 40 percent in professional services. We return to the sources of these differential sectoral responses below.

For the effects of interpersonal skill composition shocks, whose effects are small at the sectoral level, we observe two opposing forces. A reduction in the aggregate share

[15]Note, this holds since the sector's aggregate labour productivity $Productivity_j$ can be expressed as the sum of firms' employment shares and their labour productivity as $\sum_{k \in j} \omega_{kj} Productivity_{kj}$.

[16]In this simple decomposition, the covariance captures the productivity contributions of resource allocation amongst all active firms. In the detailed analysis below we isolate the effects of firm exit and re-allocations between incumbent firms.

Exercise	Production	Prof. Services
Benchmark	0.914	1.958
-10% shock to cognitive	0.875	1.877
-10% shock to manual	0.941	2.001
-10% shock to interpersonal	0.926	1.984

(a) Mean Productivity – Composition Shocks

Exercise	Production	Prof. Services
Benchmark	0.705	1.003
-10% shock to cognitive	0.658	0.949
-10% shock to manual	0.745	1.036
-10% shock to interpersonal	0.697	1.004

(b) Covariance of Productivity and Employment – Composition Shocks

Table 6: Olley-Pakes Decomposition of Sectoral Labour Productivity: Average vs Allocation Across Firms – Composition Shocks

This table shows the Olley-Pakes decomposition of sectoral labour productivity according to (11). The first component, top panel, is the (unweighted) mean productivity at the firm level; the second component, bottom panel, is the covariance between firms' employment share and their labour productivity. Reported values are absolute values.

of interpersonal skills increases mean productivity in each sector, but it decreases the covariance term in the production sector, i.e. the extent to which larger firms have higher labour productivity declines. Once again, these results highlight the importance of distinguishing between skill types and industrial sectors when drawing conclusions about productivity.

Firm level. Since both within- and between-firm margins are important for sectoral outcomes, we now examine the underlying mechanisms at the firm level.

The skill supply shocks alter equilibrium skill wages. While these wage changes are common across firms, their effects differ because of firms' heterogeneity in idiosyncratic productivity paired with overhead labour requirements. As a result, the skill shocks affect firms' optimal skill mix, size, and profitability. In equilibrium, this generates adjustments along three margins: the composition of operations labour within firms, the distribution of employment across firms, and the selection of active firms through entry and exit.

One potentially important determinant of productivity in the average firm is the exit threshold, s^*, defined as the level of idiosyncratic productivity below which incumbent firms choose to exit. A reduction in this threshold, for instance, implies that more low productivity firms remain in the sector, which, all else equal, suppresses average and aggregated productivity. Table 7 shows how this exit threshold changes in response to skills composition shocks.

Exercise	Production	Prof. Services
Benchmark	0.783	2.247
-10% shock to cognitive	0.781	2.238
-10% shock to manual	0.777	2.246
-10% shock to interpersonal	0.795	2.269

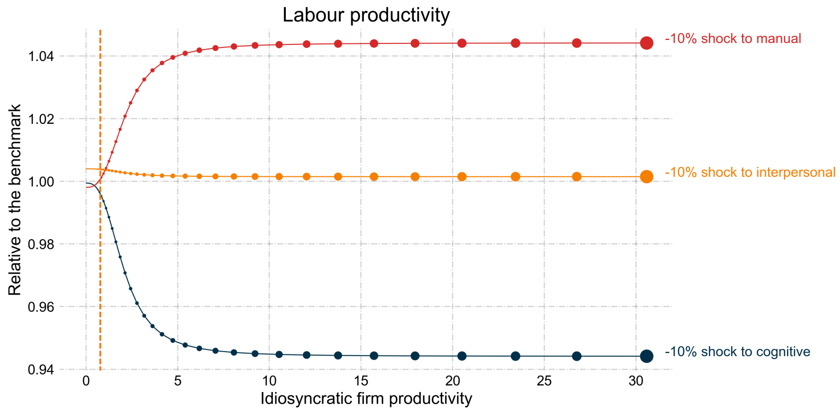
Table 7: Exit Threshold – Composition Shocks

This table shows the idiosyncratic productivity values below which incumbent firms exit the market at the end of the period, for each sector and exercise.

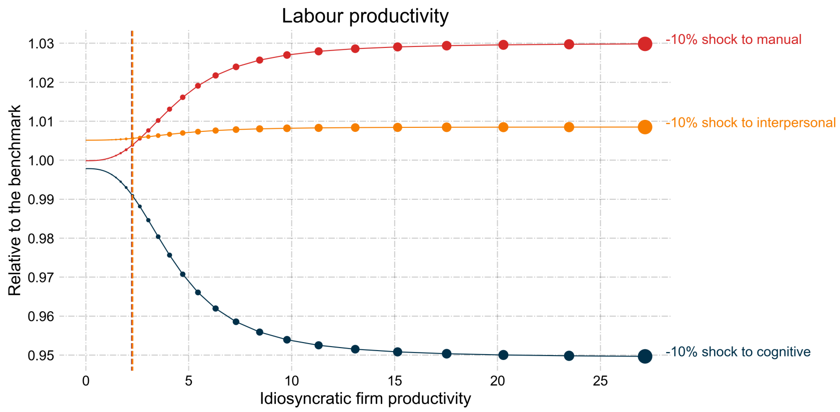
The shocks affect exit thresholds in both sectors, but the changes are quantitatively small. This suggests that selection through firm exit plays only a limited role in explaining the productivity responses documented above. Moreover, comparing Table 7 with the results for mean productivity in Table 6a, changes in the exit threshold do not consistently move in the same direction as mean productivity, indicating that selection through firm exit is not the main driver of sectoral labour productivity changes.

Instead, the main adjustment occurs through other margins, including changes in employment and skill composition within active firms, adjustments in firm entry, and shifts in the joint distribution of employment and idiosyncratic productivity among active firms.

We therefore now turn to the cross-sectional effects across active firms. We begin with examining how the skill composition shocks affect the distribution of labour productivity across firms. Recall that all within-sector heterogeneity arises from differences in firms' idiosyncratic Hicks-neutral productivity, s_k. Figure 3 plots labour productivity as a function of s_k for each skill composition shock, relative to the benchmark economy. Note that the mass of firms is not distributed uniformly across the support of s_k, but is determined endogenously by the equilibrium distribution of active firms in each sector. The dashed vertical lines indicate the exit thresholds, which change little across scenarios (as in Table 7).



(a) Production Sector

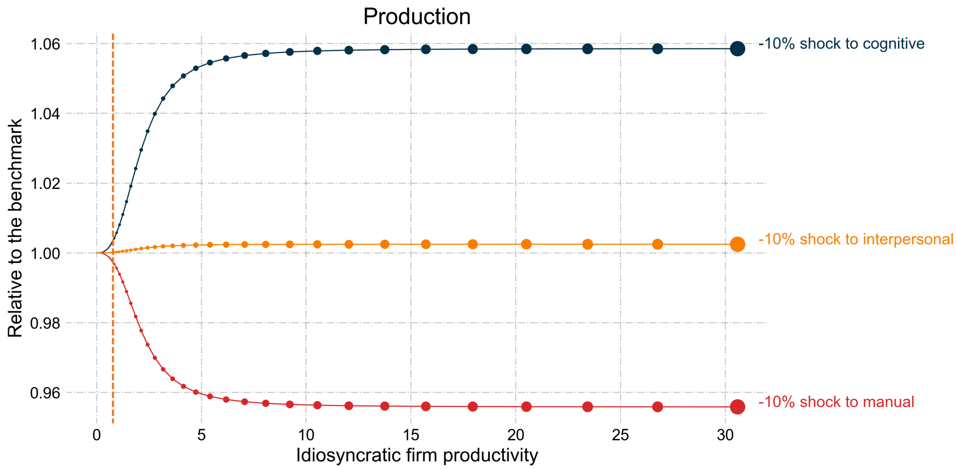


(b) Professional Services Sector

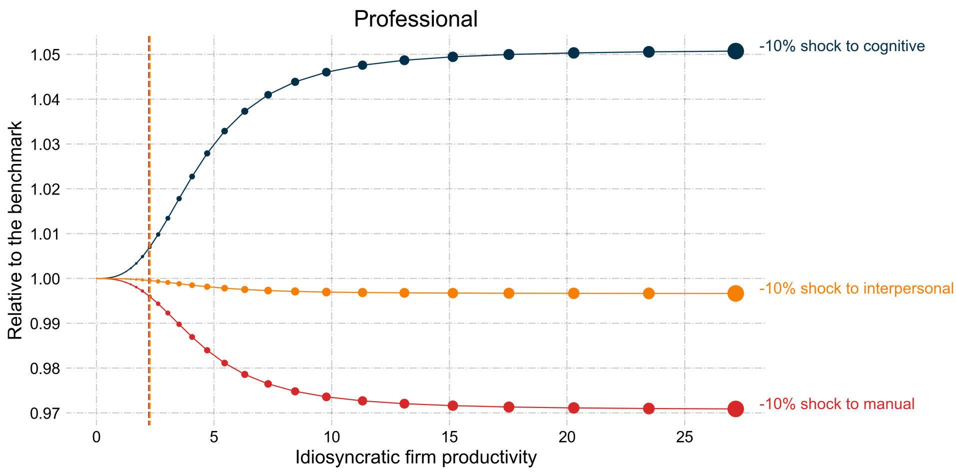
Figure 3: Labour productivity distribution across firm types – Composition Shocks

Labour productivity (of incumbent firms) for each idiosyncratic firm technology, s, in different economies relative to the benchmark economy. The dashed vertical lines represent the (sector-specific) idiosyncratic threshold s^* below which firms exit the market. Circle sizes are proportional to the total employment in incumbent firms with the corresponsing idiosyncratic productivity type.

Figure 4 plots the corresponding behaviour of firm employment as a function of firm technology in each sector.[17] As shown by (5) and (6), firms with higher technology draws are larger in terms of both output and employment, and therefore also employ more of each skill, as indicated by (4).[18]



(a) Employment Distribution - Production



(b) Employment Distribution - Prof. Services

Figure 4: Employment distribution across firm types – Composition Shocks
Incumbent firm employment for each idiosyncratic firm technology, s, in different economies relative to the benchmark economy. The dashed vertical lines represent the (sector-specific) idiosyncratic threshold s^* below which firms exit the market. Circle sizes are proportional to the total employment in incumbent firms with the corresponding idiosyncratic productivity type.

These figures demonstrate that firms' responses vary across idiosyncratic techno-

[17] Since skills intensities and wages are common to all firms within a sector, a firm's total operations employment share (within the sector) is identical to the operations employment share in each skill.

[18] The slope of the output-employment relationship across firms changes with the real unit labour cost w_j/p_j (shown in Appendix Table A4), and thus in response to skill supply shocks, see Appendix Figure A2. The change in the gradient can be understood by comparing (5) and (6), noting that $0 < \theta < 1$.

logy realizations and thus by firm size. For instance, the negative manual skill composition shock increases labour productivity in both sectors, as reported in Table 5, but the gains are concentrated among the most productive firms. Conversely, a reduction in the cognitive skills share lowers labour productivity in all sectors and firms, but with the impact increasing in idiosyncratic firm productivity. The effects of the skill supply shocks therefore vary systematically with firm size.

The differential impacts across firms arise from the presence of overhead labour requirements. Because part of a firm's labour input is fixed, firms can adjust only their operations labour in response to skill supply shocks. Since within a sector firms differ only in their idiosyncratic productivity, a given change in relative wages has the same effect on operations labour choices for all firms. When the supply of cognitive skills falls, they become more expensive relative to the other skills (see Appendix Table A4). Since the changes in relative wages are common, all firms adjust the skill mix of operations labour in the same way, as shown in (4). This alters the ratio of output to operations labour by the same factor for all active firms within a sector. Because cognitive skills are relatively scarce in equilibrium, a reduction in their supply results in a uniform decline in the output-to-operations-labour ratio for all firms in a sector, as illustrated in Appendix Figure A1.

Yet, the effect on firms' labour productivity differs across firm sizes because labour productivity is the ratio of output to *total* employment, i.e., the *sum of operations and overhead* labour. For very large firms, overhead labour is negligible, so labour productivity closely tracks output per unit of operations labour, which is approximately equal to the ratio of output to overall employment. However, for smaller firms labour productivity is less tightly linked to output per operations worker. Changes in operations labour affect both output (the numerator in labour productivity) and totoal employment (the denominator). However, because overhead labour is fixed, the denominator responds less in smaller firms, where overhead labour constitutes a larger fraction of total employment. Consequently, labour productivity becomes less sensitive to changes in operations labour. The presence of overhead labour therefore imply that a reduction in cognitive skill supply results in a larger decline in labour productivity for large firms than for small firms.

Looking more closely at Figure 3, we see one difference between shocks to manual and interpersonal skills. In both cases, labour productivity effects are positive for all active firms ($s > s^*$). However, while typically the shocks impact larger firms' labour productivity more strongly, in the production sector a reduction in the interpersonal skills has a relatively larger effect on smaller firms.[19] Recall that labour overheads are negligible for large firms, so their productivity dynamics are driven primarily by changes in operations labour. In the professional services sector, this shock reduces operations labour in all firms by approximately 0.3%, implying larger productivity gains for large firms, where labour productivity closely tracks output per unit of operations labour. The pattern differs in the production sector. There, the reduction in the interpersonal skill share increases operations employment in all firms. As a result, in the production sector labour productivity rises by less in large firms than in small firms.

It is important to bear in mind that these graphs represent the distribution of firms by type, where firms are classified according to their technology draw. The mass of firms in each type is shaped by the exogenous productivity processes and the endogenous entry and exit dynamics that result in the stationary distributions of active firms. By applying the distribution of incumbents F_j and entrants G_j, we get the cross-sectional distributions of outcomes. Figure 5 shows the shares of firms grouped by employment bracket.

The differential behaviour of firms by size also has implications for their labour demand responses. Figure 4 already revealed that skill supply shocks affect the distribution of employment across firms within a sector. For instance, a reduction in the cognitive skill supply shifts employment towards larger firms. In large firms, which are those with a high idiosyncratic technology s, overhead labour is only a small portion of overall employment. As a result, their labour demand for the various skills is highly responsive to changes in relative wages, with an elasticity of relative skill demand that approaches the structural parameter σ that represents the elasticity of substitution in *operations labour* (2). In contrast, for firms with a low technology draws, labour demand is much less elastic. These firms are smaller and a large share of their

[19]This result also emerges under absolute skills shock, see Appendix Figure A4.

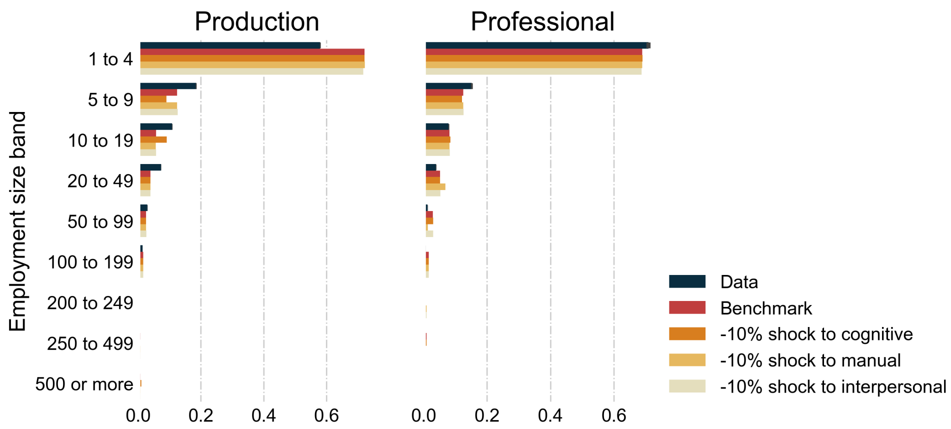
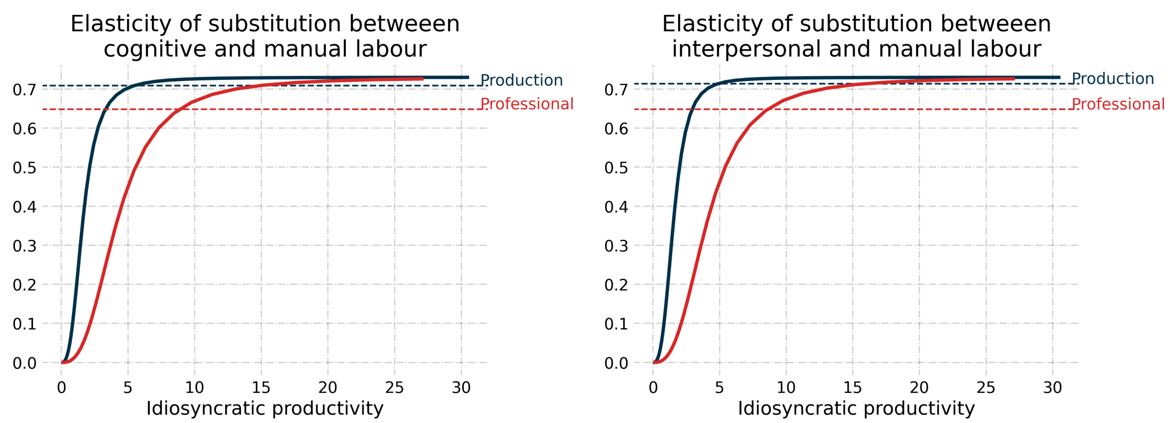


Figure 5: Business Share by Employment Size Band – Composition Shocks

Each bar shows the measure of firms (both incumbent and entrant) with the number of employees in that band relative to the total measure in the sector. The data bars are calculated using Business Population Estimates dataset. Since employment in the data is discrete, but continuous in the model, firms with employment between two consecutive labels are counted in the first label. For example, the 1 to 4 band contains firms with employment in the range $[1, 5)$.

employment is tied to overhead labour, which cannot be substituted. For these firms, the elasticity of substitution between skills in *total labour demand* is much smaller than σ.



(a) Cognitive vs Manual

(b) Interpersonal vs Manual

Figure 6: Derived Elasticities of Substitution – Composition Shocks

Notes: Solid lines show the firm-level elasticity of substitution for incumbents; dashed lines show the implied industry-wide average.

In Figure 6, we plot the derived substitution elasticities across skill types, computed from our quantitative model, against the firms' idiosyncratic technology. In both sectors, the derived firm-level relative labour demand elasticity increases monotonically, ranging from 0 to 0.73. The lowest value applies to firms that are so small (due to low s) that –if they are operating– they virtually hire only overhead labour. The upper limit to which the derived elasticity converges is 0.73, which is the value of the structural elasticity of substitution σ for operations labour in (2). This is the elasticity for firms that are so large that overhead labour accounts for a negligibly small share of total employment. For any given value of idiosyncratic technology, the elasticity is lower in professional services, the sector with the higher overhead labour requirement (see Table 2b for $C_{f,j}$). This is one of the key factors driving cross-sector differences in the response to shocks in the skills composition.

Reallocations between firms. The heterogeneous responses across firms also generate reallocations that contribute to sectoral productivity changes. These effects of these reallocations are captured by the covariance component of the Olley-Pakes decomposition reported in Table 6b. Depending on the skill shock, employment may shift toward larger or smaller firms, thereby altering the relationship between firm size and productivity.

Consider the negative cognitive composition skills shock. Since cognitive skills are relatively scarce, the shock lowers labour productivity in all firms. However, because overhead labour is less important for larger firms, the decline in labour productivity is stronger among larger firms. Figures 3 and 4 show that firms with higher idiosyncratic technology are getting larger in terms of employment, but experience a disproportionately large reduction in labour productivity.

As a result, the positive relationship between firm size and labour productivity weakens, causing the covariance term in the Olley-Pakes decomposition to fall. Sectoral productivity therefore declines not only because average firm productivity falls, but also because after the shock larger firms are now less productive. The firm-level patterns shown in Figures 3 and 4 are thus the microeconomic counterpart of the changes in the allocation component reported in Table 6b.

Taking stock. Shocks to the skills supplies systematically affect labour productivity. A reduction in the supply of cognitive skills lowers labour productivity in all sectors and firms, indicating that cognitive skills are relatively scarce in equilibrium. By contrast, reductions in manual skills increase productivity, suggesting that manual skills are relatively abundant, while interpersonal skills are moderately so.

The adverse effects of reducing cognitive skills operate through several channels. First, the shock raises the relative price of cognitive skills and induces all firms to substitute away from them in operations labour. As a result, the ratio of output to operations labour declines uniformly across firms within a sector (Figure A1). Second, because firms differ in the importance of fixed overhead labour, the resulting changes in labour productivity vary systematically with firm size. Larger firms experience larger productivity declines, which weakens the positive relationship between firm size and productivity and lowers the allocation component of sectoral productivity. Changes in exit thresholds are comparatively small, indicating only a limited direct role for firm exit. In contrast, shocks to manual skills lead to the opposite pattern.[20]

Overall, a reduction in cognitive skill supply lowers labour productivity both by reducing productivity within firms and by altering the within-sector firm distribution. The latter occurs because firms with high idiosyncratic technology draws grow even larger but experience relatively larger labour productivity declines, weakening the positive association between firm size and productivity.

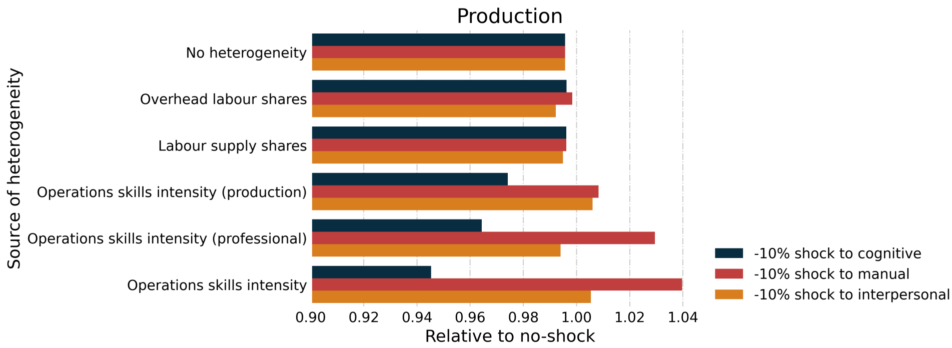
There are also important sectoral differences. Interpersonal skill shocks, for example, have differential productivity effects across the firm-size distribution in the two sectors. Moreover, the relative importance of reallocation across firms differs across sectors. For cognitive skill composition shocks, changes in the allocation across firms account for around 55 percent of the total labour productivity effect in the production sector, compared with only around 40 percent in professional services.

5.2 Understanding the causes of differential effects by skill

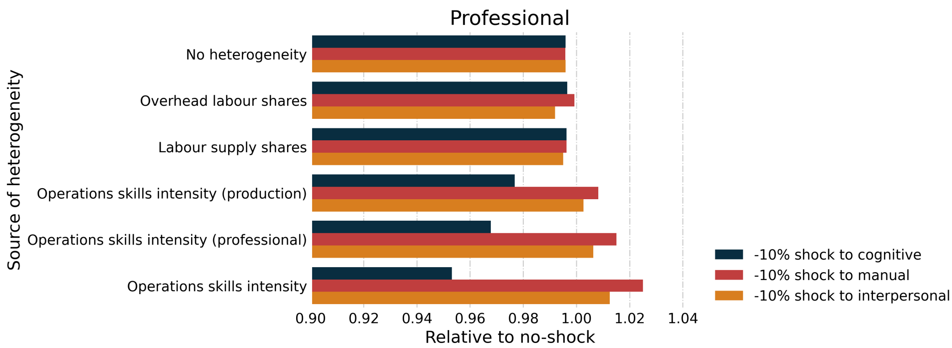
To better understand the sectoral differences and identify the origins of the differential effects across skill shocks, we conduct a series of counterfactual exercises that remove elements of skill heterogeneity from the model.

Departing from our baseline calibration (Table 2b), we impose (i) homogeneous op-

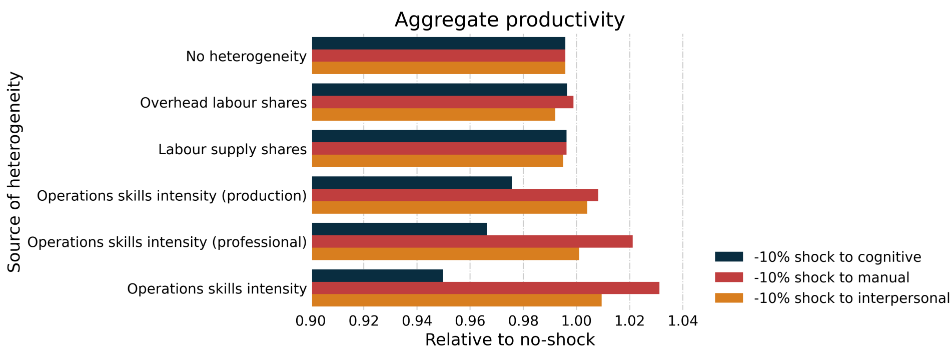
[20]By and large, the effects of interpersonal skills shocks resemble those of manual skills, although some impacts are minimal and, in a few instances, with the opposite sign.



(a) Production Sector Labour Productivity



(b) Professional Services Labour Productivity



(c) Aggregated Labour Productivity

Figure 7: Effect Heterogeneity by Source – Composition Shocks

In these exercises we begin by removing all skills heterogeneity from the model and introduce one type of heterogeneity at a time. For each type of heterogeneity we conduct three exercises where we shock the supply of each skill, one at a time. We then calculate the sectoral and aggregate productivity in each exercise relative to the corresponding productivity in the ‘no-shock’ case. Note that the ’no-shock’ case is not the benchmark economy but it refers to the economy with the corresponding heterogeneity but no skill supply shock. In the last three experiments, we vary the ‘operations skills intensity’, first only in the production sector ‘(production)’, then only in professional services ‘(professional)’, and finally in both sectors.

erations skills intensities across the three skills i and all sectors j by setting $\phi_{i,j} = 1/3$, (ii) identical skill intensities in overhead labour $n_{f,i} = 1/3$, and (iii) identical aggregate skills supply $N_i = 0.2$. Imposing (i), (ii), and (iii) simultaneously results in perfectly symmetric skills. The economy's two sectors continue to differ through their idiosyncratic productivity processes for incumbent and entrant firms, but these differences are 'skill-neutral' in the sense that they affect only the firm-size distribution and not the skill composition of employment. We then reintroduce skill heterogeneity one feature at a time.

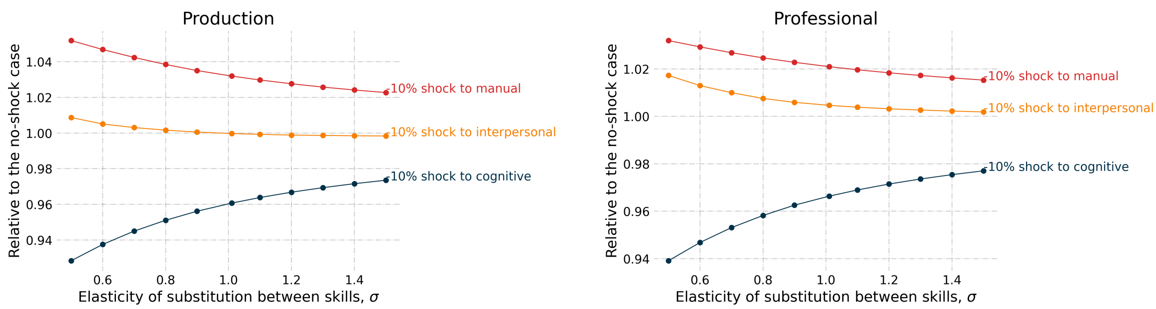
Figure 7 shows the results from this exercise.[21] By far, most of the differences across skills shocks originate from sectoral differences in operations skills intensities. The effect of introducing this element of heterogeneity dwarfs all other sources. Differences in skills intensities in overhead labour or in the aggregate skills supplies (by themselves) have only modest impacts on the economy's response to skill composition shocks. In contrast, heterogeneity in operations skills intensities across skills and sectors (the last set of bars in Figure 7) accounts for nearly all of the benchmark model's response to the cognitive skills shock. When we remove the differences in operations skills intensities from just one sector at a time (the two penultimate sets of bars), the response to skill supply shocks is attenuated, but remains substantial.

This finding is intuitive. All firms compete for the same aggregate pool of skills used in operations labour. If a particular skill, such as cognitive skills, is already relatively scarce because sectoral demand patterns differ from the aggregate skill supply composition, a further reduction in its supply only exacerbates that scarcity. The resulting labour productivity effects arise regardless of which sector initially generates the imbalance. At the same time, the results show that when there is heterogeneity in operations skills intensities across sectors, the impact on labour productivity is more pronounced.

[21] Alternatively, the results can be reorganized by skill shocks, as shown in Appendix Figure A3.

5.3 Sensitivity and Robustness

Here we briefly show that our results are robust across some of the parameters we fixed outside the calibration.



(a) Production Sector

(b) Professional Services

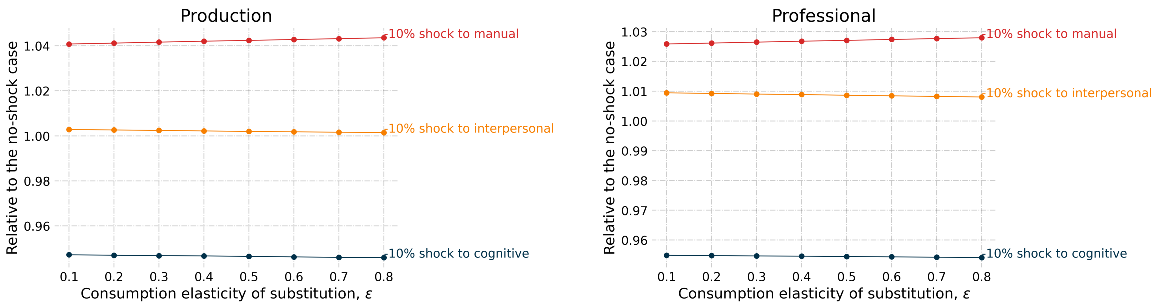
Figure 8: Comparative Statics: Role of σ for Sectoral Labour Productivity – Composition Shocks

This graph shows labour productivity for alternative values of the elasticity of substitution between skills in production, σ. The benchmark parametrisation is $\sigma = 0.73$.

Figure 8 shows how the model's response to skills shocks varies with the elasticity of substitution between skills in operations labour. Our baseline value is $\theta = 0.73$, based on Adachi (2025). This is a plausible value for the firm level elasticity where we expect the various skills to be complements, so $\theta < 1$. Nonetheless, we also consider much higher values including the range where the three skills are substitutes (as often found at the macro-level). We can see that higher values somewhat mute the responsiveness of labour productivity to skill shocks, but they do not change fundamentally, not even when skills become substitutes (i.e., $\theta > 1$).

In Figure 9 we plot how the effects of the skills supply shocks depend on the elasticity of substitution between sectoral consumption goods, ε. We see this value plays almost no role for the quantitative results.

In Appendix Section E, we report several additional robustness exercises. First, we conduct a sensitivity analysis with respect to the persistence parameter ρ of firms' idiosyncratic productivity process. As Figure A6 shows, the labour productivity effects of skill supply shocks are almost identical across values of ρ and reductions in cognitive skills continue to generate large negative effects on labour productivity in both sectors.



(a) Production Sector (b) Professional Services

Figure 9: Comparative Statics: Role of ε for Sectoral Labour Productivity – Composition Shocks

This graph shows labour productivity for alternative values of the elasticity of substitution between sectoral consumption goods, ε, on the horizontal axis. The benchmark parametrisation is $\varepsilon = 0.2$.

Second, we allow the skill composition of overhead labour requirements to differ across sectors by extending the definition of overhead labour beyond managerial tasks to include other administrative occupations. As shown in Figure A7, the productivity effects of cognitive-skill shortages remain very similar to those in the baseline calibration.

Third, we allow substitution elasticities to differ across skill pairs. Table A12 reports the effects of a reduction in cognitive skills. As expected, greater substitutability mitigates the negative productivity consequences of the shock, but the qualitative results remain unchanged.

Finally, Figure A8 compares the productivity effects of a 10% reduction in cognitive skills under three alternative shock specifications: a skill-composition shock (our baseline), an absolute skill supply shock, and a correlated composition shock in which cognitive and interpersonal skills move together. The results show that allowing cognitive and interpersonal skills to be correlated strengthens the productivity consequences of cognitive-skill shortages, while leaving the qualitative ranking of skills unchanged.

6 Conclusions

Our findings underscore the importance of accounting for the multi-dimensional nature of skills shortages. Our model-based analysis suggests that, in the current UK economy, a shortfall in cognitive skills is holding back labour productivity. That is, cognitive skills are relatively scarce, whereas manual skills are in abundance, and interpersonal skills moderately so.

One implication is that aggregate metrics on worker skills may be misleading in debates about overall productivity. Identifying bottlenecks requires measuring skills along multiple dimensions. Our analysis points to cognitive skills as the critical constraint. Similarly, aggregate productivity measures may obscure substantial heterogeneity in how firms and sectors are affected by skill shortages.

Indeed, we find that incorporating sectors in the analysis is important, as sectoral variation in operations skills intensities amplifies the impact of cognitive skill shortages, not only in the aggregate but across all sectors.

Furthermore, our general equilibrium model with firm dynamics reveals that a shortage of cognitive skills depresses labour productivity in two ways: (i) by reducing average firm-level productivity, as the operational skill mix shifts away from the scarce cognitive skill, and (ii) by distorting the within-sector firm size distribution, whereby larger firms grow in relative size but become less productive.

To our knowledge, this second channel has not been emphasised in the existing literature and emerges directly from the model we develop in this paper. It underscores the importance of incorporating firm dynamics into the analysis of skill shortages. Therefore skills shortages may play an even larger role in limiting aggregate productivity than previously recognised.

Our findings also have implications for policy. Our quantitative model identifies cognitive skills as the most important skill margin for labour productivity. This implies that investments aimed at expanding cognitive skills may yield larger aggregate productivity gains than equivalent investments in other skill dimensions.

In the UK context, this conclusion is broadly consistent with recent policy initiatives such as the National Skills Fund, Skills Bootcamps, and the forthcoming Lifelong Learning Entitlement. All of these programmes seek to improve the alignment between labour market demand and the composition of workforce skills by expanding opportunities for adult retraining and upskilling. Our findings suggest that policies supporting the acquisition of cognitive skills, including analytical and numeracy competencies, may be particularly effective in raising labour productivity in the UK economy. They also have implications for earlier stages of skill formation. School curricula that place greater emphasis on quantitative reasoning and problem solving may increase long-run productivity by shifting the composition of workforce skills towards cognitive competencies.

As our model abstracts from the costs of skill acquisition, heterogeneity in workers' ability to retrain, and transitional adjustment dynamics, these results should not be interpreted as a direct evaluation of particular policy interventions. Rather, they highlight the macroeconomic importance of the composition of workforce skills.

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Appendix

A Data

Industry Code/Label	Industry Group
A Agriculture, forestry and fishing	Production
B Mining and quarrying	Production
C Manufacturing	Production
D Electricity, gas, air cond supply	Production
E Water supply, sewerage, waste	Production
J Information and communication	Professional services, technology and media
L Real estate activities	Professional services, technology and media
M Prof, scientific, technical activ.	Professional services, technology and media
F Construction	Construction
G Wholesale, retail, repair of vehicles	Trade, transportation and hospitality
H Transport and storage	Trade, transportation and hospitality
I Accommodation and food services	Trade, transportation and hospitality
K Financial and insurance activities	Finance
N Admin and support services	Other Services
O Public admin and defence	Government, education and healthcare
P Education	Government, education and healthcare
Q Health and social work	Government, education and healthcare
R Arts, entertainment and recreation	Other Services
S Other service activities	Other Services

Table A1: Mapping of Industry Labels to Industry Groups

Dataset	Sample Period
Business Population Estimates for the United Kingdom (BPE)	2013-2023
Business demography, quarterly, UK Statistical Bulletins	2017-2022
Business demography, annual, UK, all firms	2009-2022
Business demography, annual, UK, employer firms	2014, 2020, 2022
UK National Accounts, The Blue Book	1990-2021
Annual Business Survey (ABS)	2000-2021
Labour Force Survey (LFS)	2011-2020

Table A2: Sample periods of the datasets in the calibration

B Comparison to Skill Premia in the Data

As an additional validation exercise, we compare the model-implied wage rates for each skill to skill premia in the data. Importantly, these wage moments are not targeted in the calibration and therefore provide an external check on the model's implications regarding the relative scarcity of skills.

Using UK Labour Force Survey data for 2011–2020, we construct an occupation-year panel containing mean hourly wages and employment shares for each occupation. We combine these data with the occupation-level cognitive, interpersonal, and manual skills measures of Lise and Postel-Vinay (2020).

In this occupation-year panel we estimate the following regression of log occupational wages, where we weight observations by their relative employment share within a year:

$$\log w_{o,t} = \alpha + \beta_C y_o^{cog} + \beta_I y_o^{int} + \beta_M y_o^{man} + \gamma_t + \varepsilon_{o,t},$$

where $w_{o,t}$ denotes the mean hourly wage in occupation o and year t, y_o^{cog}, y_o^{int}, and y_o^{man} denote the corresponding skill intensities, and γ_t are year fixed effects. Standard errors are clustered at the occupation level. Table A3 reports the results.

The estimated coefficients imply a clear ranking of skill premia. Cognitive skill intensity is associated with the highest wages, followed by interpersonal skills, while manual skills receive substantially lower relative compensation conditional on the other skill dimensions.

To facilitate comparison with the quantitative model, we compute implied relative skill prices from the estimated coefficients. Specifically, the interpersonal-to-cognitive and manual-to-cognitive skill-price ratios are given by

$$\frac{w_I}{w_C} = \exp(\beta_I - \beta_C), \qquad \frac{w_M}{w_C} = \exp(\beta_M - \beta_C).$$

The estimated ratios are 0.394 and 0.190, respectively. These values are remarkably close to the corresponding model-implied wage ratios of 0.415 (interpersonal relative to cognitive) and 0.171 (manual relative to cognitive), as can be seen in Table A4.

	Log Wage
Cognitive skill intensity	1.50***
	(0.15)
Manual skill intensity	-0.16
	(0.13)
Interpersonal skill intensity	0.57***
	(0.12)
Constant	1.65***
	(0.10)
Year FE	yes
Observations	2,828
R^2	0.69
Interpersonal / Cognitive Skill Price	0.394
Manual / Cognitive Skill Price	0.190

Standard errors clustered at the occupation level in parentheses.
Observations weighted by occupation employment shares within each year.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table A3: Wage regression on skill intensities

Since the calibration does not target wage premia across skill dimensions, the ability of the model to reproduce both the ranking and the magnitude of relative skill prices provides additional support for empirical relevance of our quantitative model.

C Derivations

Firms' optimization We can break down an incumbent firm's problem into a static and dynamic part. In the following we focus on an individual firm and therefore drop firm (k) and sector (j) indices. Future value are denoted by a prime ($'$).

In the static part, firms minimize variable costs. This entails choosing the amount of each skilled labour input for a given composite labour L as described by (2):

$$\min \sum_i w_i n_i \text{ s.t. } L = \left(\sum_i \phi_{j,i}^{1/\sigma} n_i^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}, \text{ where } i \in \{C, M, I\}.$$

Denoting the Lagrange multiplier by λ, the first order conditions require

$$w_i = \lambda L^{1/\sigma} \phi_{j,i}^{1/\sigma} n_i^{-1/\sigma} \text{ for } i \in \{C, M, I\}.$$

or after rearranging

$$n_i = \lambda^\sigma L \phi_{j,i} w_i^{-\sigma} \text{ for } i \in \{C, M, I\}.$$

Substituting this into (2) solves for the Lagrange multiplier, which due to the properties of the cost minimization problem, equal to the unit cost of composite labour w

$$w_j \equiv \lambda = \left(\sum_i \phi_{j,i} w_i^{1-\sigma} \right)^{1/(1-\sigma)}. \tag{12}$$

This composite labour unit cost index w is such that

$$\sum_i n_i w_i = w_j L. \tag{13}$$

A firms' optimal variable demand for each skill i satisfies

$$n_i = \phi_{j,i} \left(\frac{w}{w_i} \right)^\sigma L \text{ for } i \in \{C, M, I\}. \tag{14}$$

The skill shares in the labour mix of a firm therefore reflects relative wages for each skill (w_i) and skill intensities ($\phi_{j,i}$), which vary across sectors (j).

The second part to the static optimization by firms is to choose the labour composite L, pinning down the firm's overall size in terms of employment. Note, in our model this is not a dynamic decision making problem, as L is not a state variable (in the absence of any labour adjustment costs) — profits in the subsequent period, and thus the continuation value in the value function, do not depend on its current value. The optimal amount of composite labour L is therefore the one that maximise flow profits in each period. Given (13), it solves $\max_L p_j s L^\theta - C_{f,j} \sum_i w_i n_{f,i} - w_j L$, requiring

$$L = \left(\frac{p_j s \theta}{w_j} \right)^{\frac{1}{1-\theta}}, \tag{15}$$

in turn this implies for firm output (1)

$$y = s^{\frac{\theta}{1-\theta}} \left(\frac{p_j \theta}{w_j} \right)^{\frac{\theta}{1-\theta}}. \tag{16}$$

Since within a sector, only s varies across firms, and $0 < \theta < 1$, firms with higher technology draws have in equilibrium larger employment and value-added shares compared to others in their sectors.

The dynamic part in the firm optimization problem stems from deciding whether to exit the market at the end of the period, before next period's productivity has been realized. This dynamic problem can be represented by the following value function

$$v(s, p_j, w_j, \{w_i\}) = \max_L p_j s L^\theta - C_{f,j} \sum_i w_i n_{f,i} - w_j L + \beta \max\{0, E_s v(s', p'_j, w'_j, \{w'_i\})\},$$

where w_j is the unit cost of labour in industry j (as defined in (13)), p_j the price of the homogenous good in sector j, w_i is the wage rate for skill type i, and $n_{f,i}$ is the share of skill type i in overhead labour. If a firm decides to exit the market, it receives a scrap value of 0, if it decides to stay the continuation value $E_s v(s', p'_, w'_j, \{w'_i\} j)$ where the expectation is conditional on the current productivity s. Under the $AR(1)$ process for firm productivity (3), $E_s v(s', p'_j, w'_j, \{w'_i\})$ is an increasing function in s, that is the expected future value of the firm increases in current productivity. Because of the fixed overhead labour costs $C_{f,j} \sum_i w_i n_{f,i}$, flow profits are negative for firms that are relatively small. Given the results of the static optimization, small firms making flosses are those with low s. Therefore, $E_s v(s', p'_j, w'_j, \{w'_i\})$ is not only monotonically increasing in s, but also negative for small s and positive only for sufficiently high s. Firms with productivity $s < s^*$ exit the market where the critical productivity is such that $E_{s^*} v(s') = 0$ leave, while firms with $s \geq s^*$ stay.

D Additional Results

Exercise	Cognitive	Manual	Interpersonal
Benchmark	1	0.171	0.415
-10% shock to cognitive	1	0.136	0.327
-10% shock to manual	1	0.215	0.414
-10% shock to interpersonal	1	0.171	0.527

(a) Wages

Exercise	Production	Prof. Services
Benchmark	0.449	0.257
-10% shock to cognitive	0.412	0.236
-10% shock to manual	0.464	0.262
-10% shock to interpersonal	0.476	0.275

(b) Sectoral Output Prices

Exercise	Production	Prof. Services
Benchmark	0.596	0.647
-10% shock to cognitive	0.547	0.596
-10% shock to manual	0.616	0.66
-10% shock to interpersonal	0.630	0.69

(c) Sectoral Unit Labour Cost (w_j)

Exercise	Production	Prof. Services
Benchmark	1.326	2.519
-10% shock to cognitive	1.326	2.522
-10% shock to manual	1.327	2.519
-10% shock to interpersonal	1.323	2.512

(d) Real Sectoral Unit Labour Cost (w_j/p_j)

Table A4: Wages and Prices – Composition Shocks

Note, $w_c = 1$ by normalization.

	Cognitive	Manual	Interpersonal
Production			
Benchmark	2.225	0.380	0.924
Composition shock	2.426	0.329	0.794
Absolute shock	2.361	0.344	0.834
Professional services, technology and media			
Benchmark	3.894	0.664	1.617
Composition shock	4.232	0.574	1.385
Absolute shock	4.124	0.601	1.456

Table A5: Real Sectoral Labour Costs (w_i/p_j) after Cognitive Skill Shocks

This table shows wages for each skill divided by sectoral output prices, w_i/p_j, where w_i is the price of skill i and p_j is the price of sector j output. The prices are shown in the benchmark economy, after a composition shock to cognitive skills, and after an absolute shock to cognitive skills.

Exercise	Production	Prof. Services
Benchmark	0.255	0.345
-10% shock to cognitive	0.256	0.344
-10% shock to manual	0.252	0.348
-10% shock to interpersonal	0.256	0.344

(a) Overall Employment – Composition Shocks

	Production Employment		Overhead Employment	
Exercise	Production	Prof. Services	Production	Prof. Services
Benchmark	0.241	0.302	0.014	0.043
-10% shock to cognitive	0.243	0.303	0.013	0.041
-10% shock to manual	0.238	0.303	0.015	0.045
-10% shock to interpersonal	0.242	0.301	0.014	0.043

(b) Production vs Overhead Employment

Table A6: Sectoral Employment – Composition Shocks

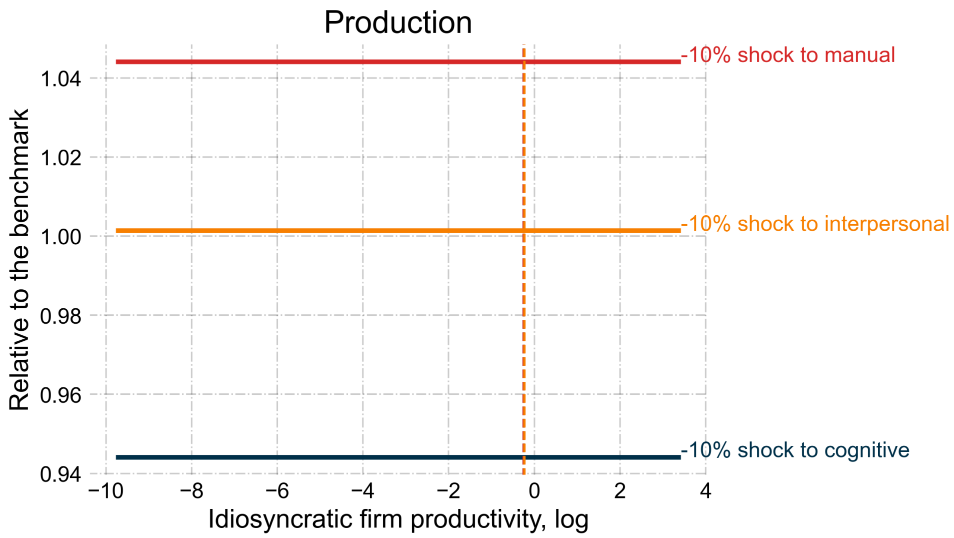
Exercise	Production	Prof. Services
Benchmark	0.975	1.019
-10% shock to cognitive	0.969	1.023
-10% shock to manual	0.991	1.007
-10% shock to interpersonal	0.965	1.026

Table A7: Relative Sectoral Revenue Productivity – Composition Shocks

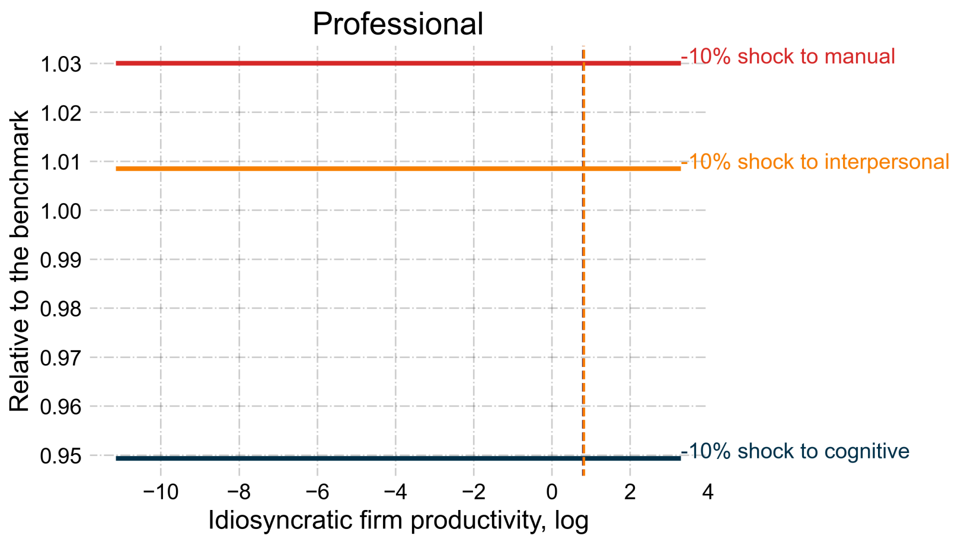
Revenue productivity in industry j over the revenue productivity in the economy.

Exercise	Production	Prof. Services
Benchmark	20.875	11.149
-10% shock to cognitive	22.004	11.618
-10% shock to manual	19.926	10.861
-10% shock to interpersonal	21.235	11.183

Table A8: Mean Firm Employment – Composition Shocks



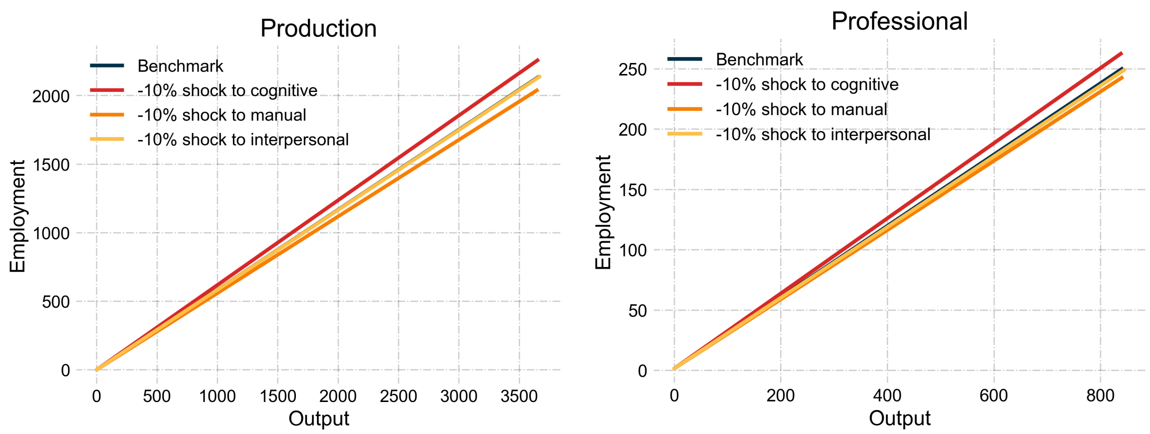
(a) Production Sector



(b) Professional Services Sector

Figure A1: Output per unit of production labour: distribution across firm types – Composition Shocks

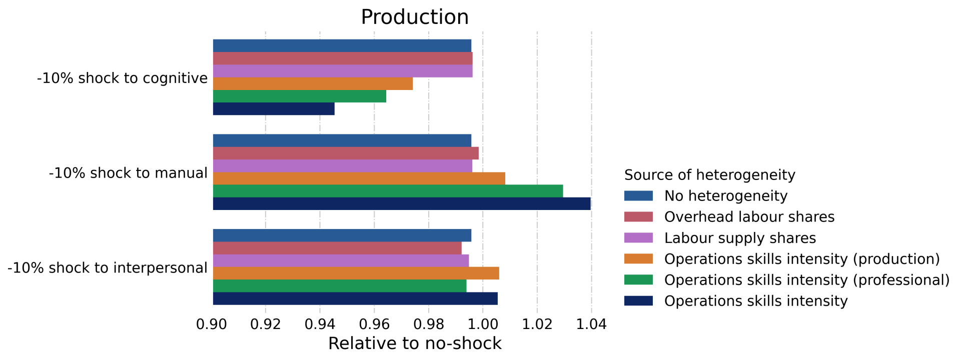
Output per unit of production labour (as opposed to labour productivity defined as output per unit of overall labour, the sum of production and overhead labour) for each idiosyncratic firm technology, s, in different economies relative to the benchmark economy. The dashed vertical lines represent the (sector-specific) idiosyncratic threshold s^* below which firms exit the market.



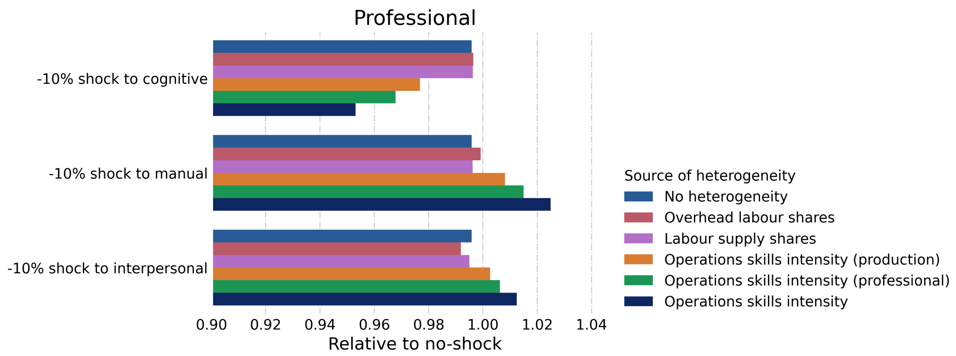
(a) Production Sector

(b) Prof. Services

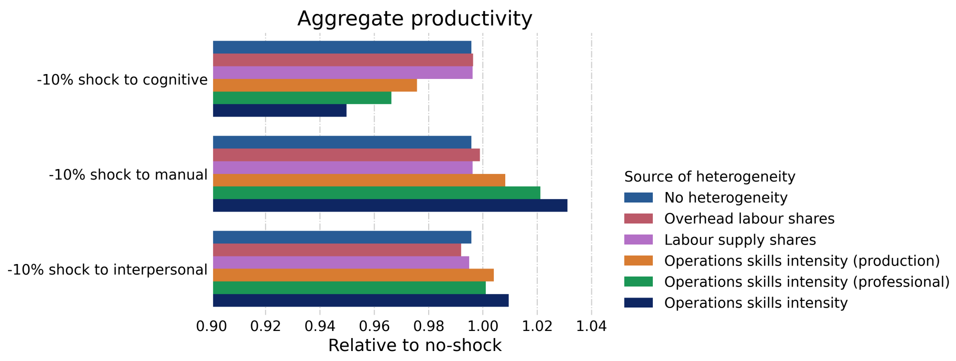
Figure A2: Employment-Output Distribution across Firms – Composition Shocks



(a) Production Sector Labour Productivity



(b) Professional Services Labour Productivity

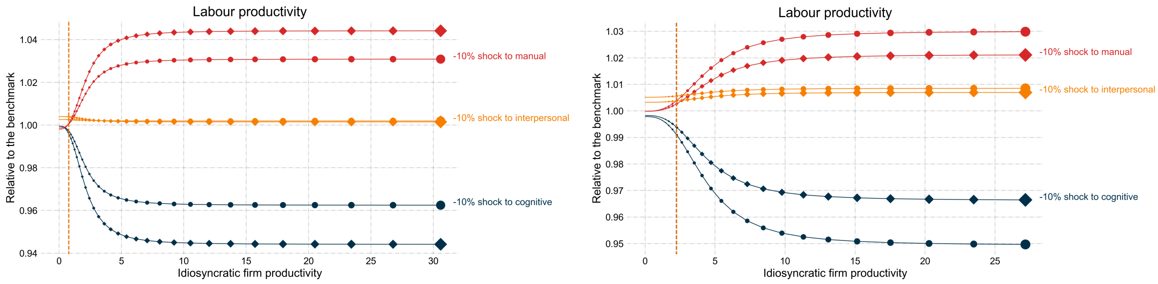


(c) Aggregated Labour Productivity

Figure A3: Effect Heterogeneity by Skills Shock – Composition Shocks

D.1 Absolute Shocks

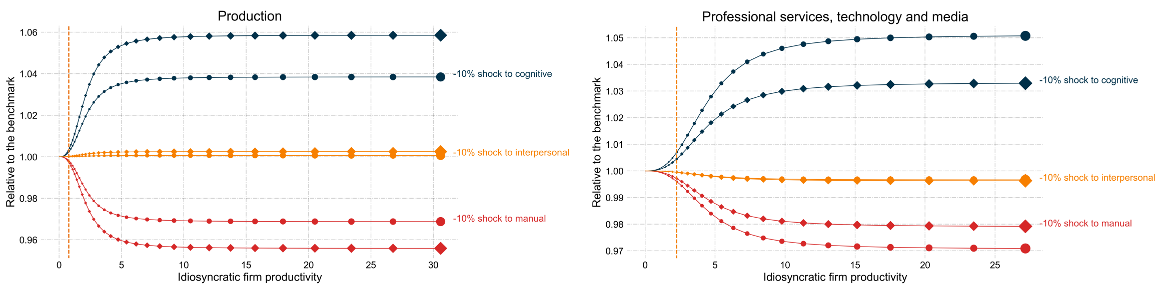
Figures A4 and A5 show the productivity and employment distribution of firms after absolute skill shocks where the supply of each skill reduced by 10% individually without an effect on the supply of other skills. Table A9 shows the Olley-Pakes decomposition results after absolute shocks.



(a) Production Sector (b) Prof. Services

Figure A4: Productivity Distribution across Firms – Absolute Shocks v. Composition Shocks

Notes: Values relative to benchmark. Absolute shocks (circles) reduce supply of one skill by 10%. Skill composition shocks (diamonds) reduce one skill by 10% while increasing the remaining skills so that total labour supply remains constant.



(a) Production Sector (b) Prof. Services

Figure A5: Employment Distribution across Firms – Absolute Shocks v. Composition Shocks

Notes: Values relative to benchmark. Absolute shocks (circles) reduce supply of one skill by 10%. Skill composition shocks (diamonds) reduce one skill by 10% while increasing the remaining skills so that total labour supply remains constant.

Exercise	Production	Prof. Services
Benchmark	0.914	1.958
-10% shock to cognitive	0.887	1.904
-10% shock to manual	0.933	1.988
-10% shock to interpersonal	0.923	1.977

(a) Mean Productivity

Exercise	Production	Prof. Services
Benchmark	0.705	1.003
-10% shock to cognitive	0.674	0.967
-10% shock to manual	0.733	1.027
-10% shock to interpersonal	0.700	1.005

(b) Covariance of Productivity and Employment

Table A9: Olley-Pakes Decomposition of Sectoral Labour Productivity: Average vs Allocation Across Firms – Absolute Shocks

This table shows the Olley-Pakes decomposition of sectoral labour productivity according to (11) after absolute skill shortage shocks. The first component, top panel, is the (unweighted) mean productivity at the firm level; the second component, bottom panel, is the covariance between firms' employment share and their labour productivity.

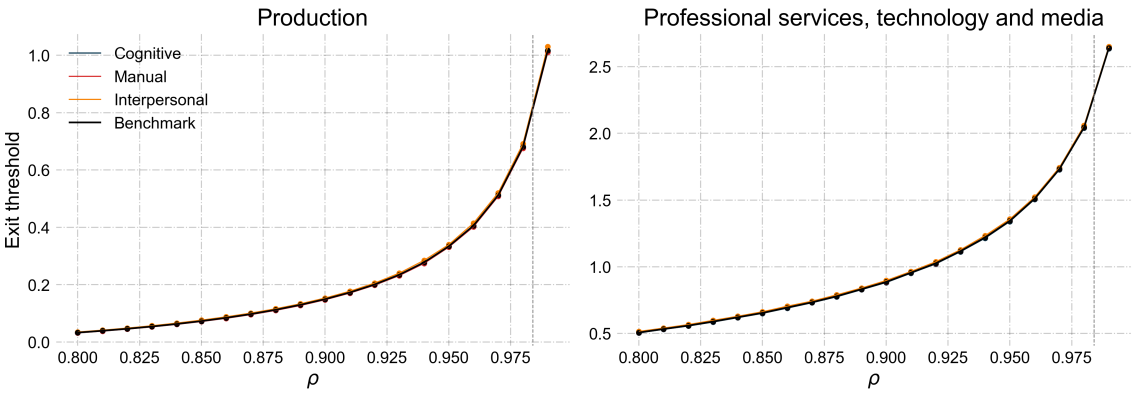
E Robustness Checks

E.1 Persistence of idiosyncratic firm productivity

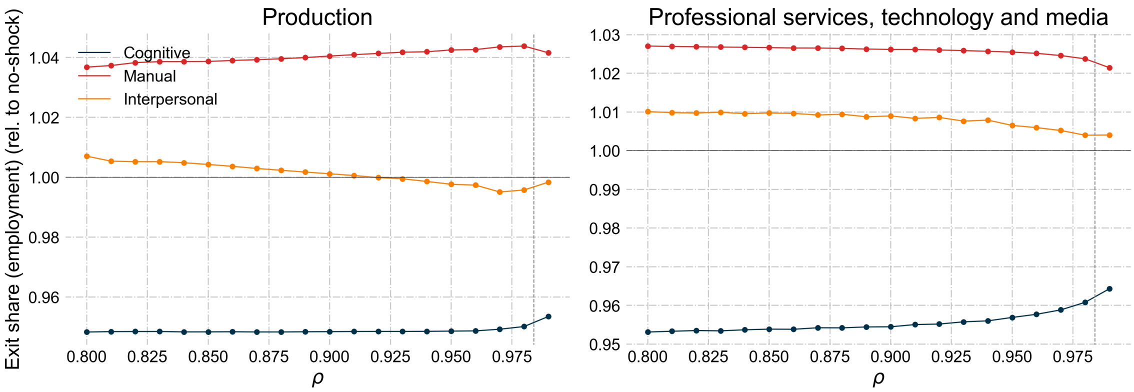
In this subsection, we conduct a sensitivity analysis with respect to the persistence parameter of the firm productivity process. Specifically, we assign alternative values of the AR(1) persistence parameter, ρ, while keeping the unconditional standard deviation of log-productivity fixed. We adjust the standard deviation of the incumbent productivity shocks, σ_i, for each ρ as follows: $\sigma_i = \sqrt{1-\rho^2}\frac{\sigma_{i,b}}{\sqrt{1-\rho_b^2}}$, where $\sigma_{i,b}$ and ρ_b are the benchmark calibration values. We keep all other parameters at their benchmark calibration levels. After setting ρ and σ_i to their corresponding values, we recompute the model equilibrium (no-shock case) and the main counterfactual exercises.

Figure A6 shows the results of these simulations against alternative values of the AR(1) productivity persistence parameter (ρ). Starting from the baseline value of $\rho = 0.984$, we consider much lower values down to 0.8 which is approximately equal to the persistence of physical TFP that Foster, Haltiwanger, and Syverson (2008) estimate from US data on continuing manufacturing establishments.

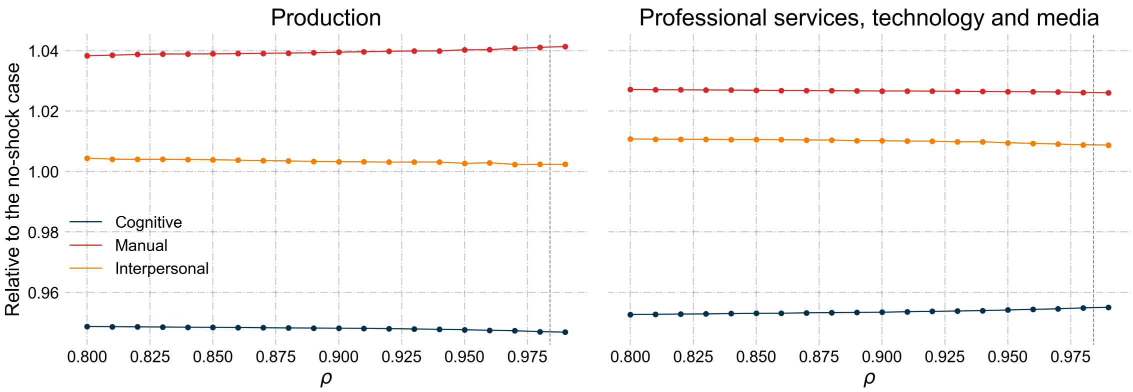
Panel A6a shows how the exit threshold is affected by ρ. Lower persistence of productivity reduces the exit threshold, i.e. reduces firm continuation incentives. This is true in the absence of shocks (note, this is a comparative static exercise, not a model recalibration) and for each of the composition shocks we simulate here. However, across the full parameter range considered, the exit margin remains quantitatively small, as the mass of firms with idiosyncratic productivity around the threshold is low. This can be seen in panel A6b which plots the employment share of exiting firms in each sector (relative to the employment share exiting firms in the corresponding no-shock economy), which is stable over AR(1) persistence parameter. As a consequence, our finding that firm exit contributes little to productivity relative to the within-firm and reallocation channels is robust to the choice of persistence of idiosyncratic productivity. Panel A6c plots the sectoral productivity changes in response to the skill composition shocks. Relative to the no-shock, productivity in each of the sectors is impacted almost identically regardless of the value of ρ, and a reduction in the cognitive skills share has throughout a strong negative effect that hardly changes with ρ.



(a) Exit threshold



(b) Employment share of exiting firms



(c) Sectoral productivity

Figure A6: Skill composition shocks against productivity persistence

E.2 Sector-specific overhead labour skill shares

In this subsection, we allow for the skill composition of overhead labour requirements to differ across sectors. We extend our definition of overhead labour beyond managerial tasks to include other tasks conducted by workers in administrative occupations (which includes secretarial tasks as this is the same 1-digit occupation code). In this extension, overhead labour shares are occupational employment weighted averages of the skill intensities of managerial and administrative occupations, where occupation weights are calculated from LFS. As the shares of these wider overhead occupations differ across sectors, this model specification makes the overhead requirements sector-specific. The overhead labour skills shares are reported in Table A10.

Model	Sector	Cognitive	Manual	Interpersonal
Benchmark (uniform)	Both sectors	0.320	0.250	0.430
Sector-specific	Production	0.340	0.257	0.403
	Prof serv, tech and media	0.353	0.234	0.413

Table A10: Skill Shares of Overhead Labour

Note. This table reports the overhead-labour skill shares in the benchmark calibration and in the alternative robustness exercise with sector-specific overhead-labour requirements. In the benchmark calibration, the shares are based on the normalized skill intensities of managerial occupations from Lise and Postel-Vinay (2020). In the sector-specific specification, the shares are constructed as occupational-employment-weighted averages of the skill intensities of managerial and administrative occupations from Lise and Postel-Vinay (2020).

When we run the model counterfactuals under this alternative, wider and sector-specific, definition of overhead labour, we get very similar results to the baseline parametrization as shown in Figure A7. The main quantitative conclusions remain robust under this alternative calibration, with productivity effects in response to skill shocks hardly changing.

E.3 Correlated cognitive skill shocks

Motivated by the multidimensional-skill literature, we now introduce an additional set of counterfactual exercises with correlated skill shocks. In these experiments, reductions in cognitive skill supply are accompanied by reductions in interpersonal skill

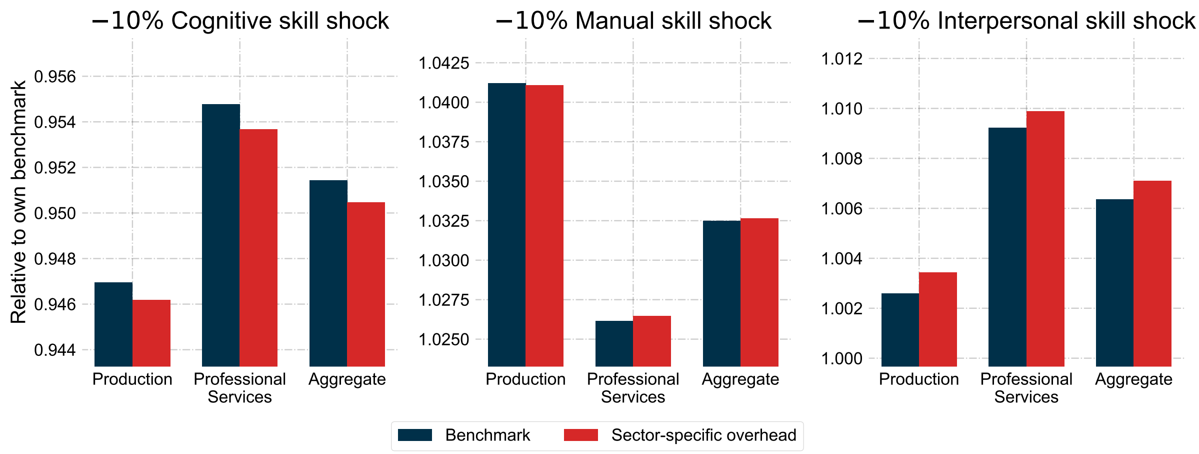


Figure A7: Productivity effects of skill composition shocks

Note. This figure shows the productivity effects of skill composition shocks under the benchmark calibration and under the sector-specific overhead skills intensities of Table A10.

supply, rather than offsetting increases. To incorporate correlated skill shocks, we first use UK Labour Force Survey data for 2011–2020 to characterize the joint distribution of skills. As throughout the paper, we construct occupational skill measures by applying the occupation-level skill intensities of Lise and Postel-Vinay (2020) to the LFS data.

Table A11 reports the pairwise correlations across the three skill dimensions; the last row additionally reports the standard deviations of each skill. Consistent with the multidimensional-skill literature, cognitive and interpersonal skills are positively correlated (while manual skills are negatively correlated with interpersonal skills).

Skill	C	I	M
C	—		
I	0.6878	—	
M	0.2401	-0.2171	—
SD	0.1730	0.2114	0.1706

Table A11: Correlations and Standard Deviations of Skill Measures

Note. The table reports pairwise correlations among cognitive (C), interpersonal (I), and manual (M) skill measures in the UK labour force. Only the lower triangle of the symmetric correlation matrix is shown. The bottom row reports the standard deviations of the marginal skill distributions.

We use these moments to construct a correlated skill-supply shocks. Specifically, when varying cognitive skills C by ΔC, we also adjust interpersonal skills I according to the linear projection

$$\Delta I = \rho_{C,I} \frac{\sigma_I}{\sigma_C} \Delta C, \tag{17}$$

where $\rho_{C,I}$ denotes the correlation between cognitive and interpersonal skills and σ_C and σ_I their standard deviations. As for our baseline shocks, we reduce the supply of cognitive skills by -10%, which is a change by $\Delta C = -0.0201$; for the correlated shock we therefore simultaneously reduce interpersonal skills by $\rho_{C,I} \frac{\sigma_I}{\sigma_C}(-0.0201) \approx 0.0169$, which is a reduction by about 8.5%. In the case of the correlated composition shock, we increase manual skills supply such that the overall skills supply remains at 0.6, in the case of an absolute correlated shock, manual skills remain unchanged at baseline.

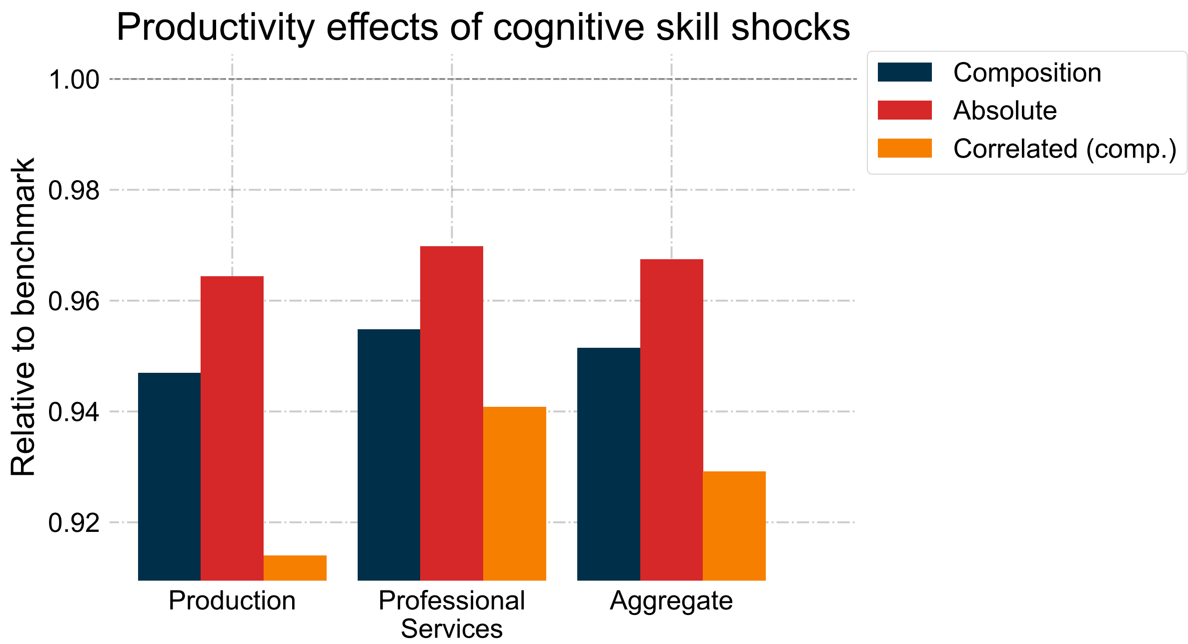


Figure A8: Productivity effects of cognitive skill shocks

This figure shows the productivity effects of a 10% reduction in cognitive skills for three alternative ways of modelling the shock: skill composition shock, absolute shock, and correlated composition shock.

Figure A8 plots the productivity effects of a 10% reduction in cognitive skills for three alternative ways of modelling the shock: skill composition change (our baseline shock), absolute shock, and correlated composition shock. These results show that for the skill composition shocks, allowing for interpersonal skills to be correlated with cognitive skills strengthens the productivity consequences of cognitive-skill shortages (for instance, aggregate productivity falls by about 7% when taking correlations into account, compared to just over 5% in the baseline), while leaving the qualitative ranking of skills unchanged. Recall that our calibration exercise finds that the cognitive skills are the most productive, then comes the interpersonal skills, and lastly the manual skills. When both cognitive and interpersonal skills supply are reduced and manual skill supply increased (correlated composition shock), firms replace high productivity skills with less productive manual skills, which then reduces productivity more than when only cognitive skill supply is reduced.

E.4 Pair-wise elasticity of substitution between skills

We now relax the common-elasticity assumption and allow the substitution elasticities to differ across skill pairs as robustness checks. To do this, we changed the production function into a nested structure. In this robustness check, the labour aggregator of firm k in industry j is now given by:

$$L_{kj} = \left(\tilde{\phi}_{j,A}^{1/\sigma_2} \left(\tilde{\phi}_{j,C}^{1/\sigma_1} n_{k,C}^{(\sigma_1-1)/\sigma_1} + \tilde{\phi}_{j,I}^{1/\sigma_1} n_{k,I}^{(\sigma_1-1)/\sigma} \right)^{\frac{\sigma_1}{\sigma_1-1}\frac{\sigma_2-1}{\sigma_2}} + \phi_{j,M}^{1/\sigma_2} n_{k,M}^{(\sigma_2-1)/\sigma_2} \right)^{\frac{\sigma_2}{\sigma_2-1}}, \tag{18}$$

where $\tilde{\phi}_{j,A} = \phi_{j,C} + \phi_{j,I}$, $\tilde{\phi}_{j,C} = \frac{\phi_{j,C}}{\phi_{j,C}+\phi_{j,I}}$, and $\tilde{\phi}_{j,I} = \frac{\phi_{j,I}}{\phi_{j,C}+\phi_{j,I}}$. In contrast, the benchmark model's $L_{kj} = \left(\sum_i \phi_{j,i}^{1/\sigma} n_{k,i}^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$.

Note, for $\sigma_1 = \sigma_2$, the specification 18 reverts back to the benchmark case with a single elasticity that is common across all skill pairs, but when $\sigma_1 \neq \sigma_2$ it implies different elasticities across skill pairs. Also note, that choosing the ordering of the CES nesting such that manual skills are in the most outer nest is in line with vom Lehn (2019), albeit here in a different context and at the firm level.

Starting from the parametrization with $\sigma_1 = \sigma_2 = 0.73$ (identical to our calibrated baseline model with a single $\sigma = 0.73$) we vary σ_1 and σ_2 one by one to alternative values of 0.63 and 0.83. For each parametrization we then simulated the effects of reducing the cognitive skill share by 10% (skill composition shock) and evaluate the productivity consequences. Table A12 reports for each parametrization productivity in the aggregate economy and in each of the two sectors following the shock relative to the productivity in the absence of a shock.

σ_1	σ_2	Aggregate	Production	Professional services, technology and media
0.63	0.63	0.945	0.940	0.949
0.63	0.73	0.947	0.943	0.951
0.63	0.83	0.949	0.946	0.952
0.73	0.63	0.949	0.944	0.953
0.73	**0.73**	**0.951**	**0.947**	**0.955**
0.73	0.83	0.953	0.950	0.956
0.83	0.63	0.953	0.947	0.956
0.83	0.73	0.955	0.950	0.958
0.83	0.83	0.957	0.953	0.960

Table A12: Productivity effects of cognitive skill composition shocks under alternative substitution elasticities

Note. This tables reports industry and the aggregate productivity after a 10% composition shock to the cognitive skill supply for various pair-wise skill elasticities. Figures are relative to the productivity levels in the no-shock case of each economy. The row in bold letters represents the benchmark economy.

Figure A9 displays the aggregate productivity effects (the third column of Table A10) for various skills-pair elasticities. As the skills get more substitutable, the decline in productivity after a cognitive composition shock decreases. To put it differently, when the skills are more substitutable, reducing the supply of cognitive skills leads to smaller productivity effects. This qualitative result holds for other skill shocks as well. With higher elasticities, the effects are smaller but still with the same sign as the benchmark exercises.

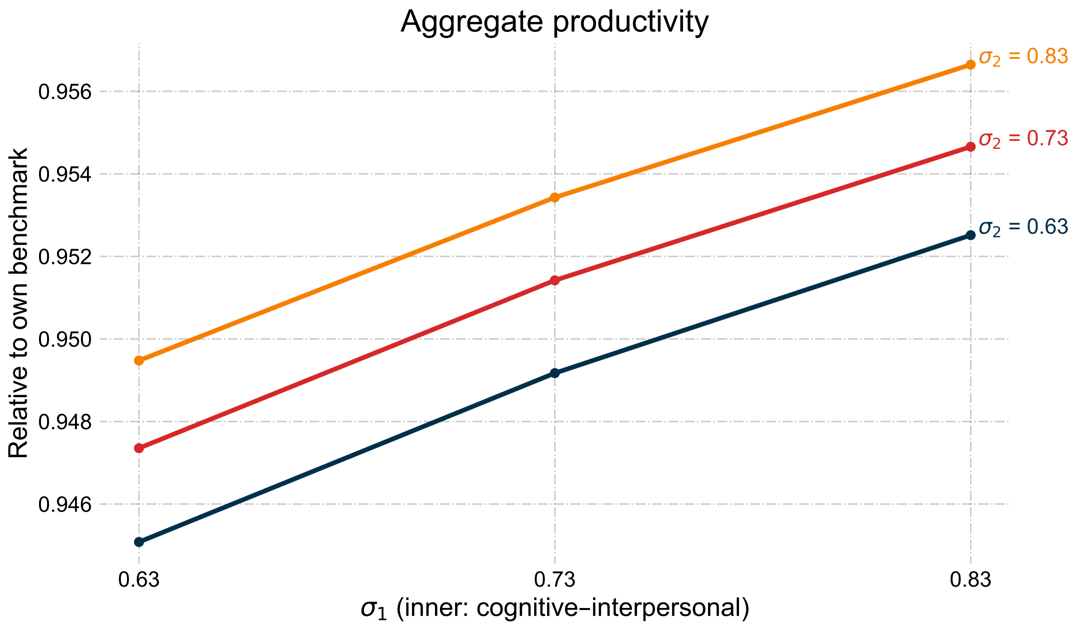


Figure A9: Productivity effects of cognitive skill shocks

F Skills grouping

Below are our manually constructed groupings of the skills found in ONS Employer Skills Survey.

Table A13: Employee Skills - Part 1 of 4

Skill	**ONS Survey Grouping**	**Our Grouping**
Instructing, teaching or training people	Soft / people	Interpersonal
Sales skills	Soft / people	Interpersonal
Customer handling skills	Soft / people	Interpersonal
Persuading or influencing others	Soft / people	Interpersonal
Team working	Soft / people	Interpersonal
Managing or motivating other staff	Soft / people	Interpersonal
Ability to manage own time and prioritise own tasks	Soft / people	Dropped from data
Setting objectives for others and planning human, financial and other resources	Soft / people	Dropped from data
Managing their own feelings, or handling the feelings of others	Soft / people	Dropped from data
Making speeches or presentations	Soft / people	Interpersonal
Interviewing	Soft / people	Interpersonal
Counselling, advising or caring for customers or clients	Soft / people	Interpersonal
Physical strength (for example, to carry, push or pull heavy objects)	Technical / practical	Manual

Table A14: Employee Skills - Part 2 of 4

Skill	**ONS Survey Grouping**	**Our Grouping**
Physical stamina (to work for long periods on physical activities)	Technical / practical	Manual
Skill or accuracy in using hands (for example, to mend, repair, assemble, construct or adjust things)	Technical / practical	Manual
Knowledge of particular products or services	Technical / practical	Cognitive
Specialist knowledge or understanding	Technical / practical	Cognitive
Reading and understanding instructions, guidelines, manuals or reports	Technical / practical	Cognitive
Writing instructions, guidelines, manuals or reports	Technical / practical	Cognitive
Number skills	Technical / practical	Cognitive
Measuring, calculating or estimating	Technical / practical	Cognitive
Spotting problems or faults	Technical / practical	Cognitive
Thinking of solutions to problems	Technical / practical	Cognitive
Analysing complex problems in detail	Technical / practical	Cognitive
Checking things to ensure there are no errors	Technical / practical	Cognitive
Following instructions precisely	Technical / practical	Cognitive

Table A15: Employee Skills - Part 3 of 4

Skill	**ONS Survey Grouping**	**Our Grouping**
Adapting to new equipment or materials	Technical / practical	Cognitive
Using or operating tools or equipment	Technical / practical	Manual
Computer literacy	IT	Cognitive
Using a computer for word processing, email etc	IT	Cognitive
Using computer applications for carrying out specialised tasks	IT	Cognitive
Writing computer software	IT	Cognitive
Designing, building, repairing computer hardware	IT	Cognitive
Building or repairing electronic equipment	Technical / practical	Cognitive
Knowledge of how different materials behave	Technical / practical	Cognitive
Knowledge of production processes	Technical / practical	Cognitive
Technical drawing	Technical / practical	Cognitive
Understanding diagrams, drawings or blueprints	Technical / practical	Cognitive
Artistic or creative skills	Technical / practical	Dropped from data

Table A16: Employee Skills - Part 4 of 4

Skill	**ONS Survey Grouping**	**Our Grouping**
Design skills	Technical / practical	Cognitive
Foreign language skills	Technical / practical	Cognitive
Driving or operating vehicles	Technical / practical	Manual
Knowledge of safety issues	Technical / practical	Cognitive
Knowledge of relevant law or regulations	Technical / practical	Cognitive
Keeping records	Technical / practical	Cognitive
Researching facts or information	Technical / practical	Cognitive
Using statistics	Technical / practical	Cognitive
Organising time or activities	Technical / practical	Dropped from data
Being able to work at speed	Technical / practical	Dropped from data
Working accurately under pressure	Technical / practical	Dropped from data
Manual skills	Technical / practical	Manual
Observing, inspecting, detecting	Technical / practical	Cognitive
Knowledge of organisation and planning	Technical / practical	Cognitive
Performing or entertaining	Technical / practical	Dropped from data

Table A17: Applicant Skills - Part 1 of 4

Skill	**ONS Survey Grouping**	**Our Grouping**
Instructing, teaching or training people	Soft / people	Interpersonal
Sales skills	Soft / people	Interpersonal
Customer handling skills	Soft / people	Interpersonal
Persuading or influencing others	Soft / people	Interpersonal
Team working	Soft / people	Interpersonal
Managing or motivating other staff	Soft / people	Interpersonal
Ability to manage own time and prioritise own tasks	Soft / people	Dropped from data
Setting objectives for others and planning human, financial and other resources	Soft / people	Dropped from data
Managing their own feelings, or handling the feelings of others	Soft / people	Dropped from data
Making speeches or presentations	Soft / people	Interpersonal
Interviewing	Soft / people	Interpersonal
Counselling, advising or caring for customers or clients	Soft / people	Interpersonal
Physical strength (for example, to carry, push or pull heavy objects)	Technical / practical	Manual

Table A18: Applicant Skills - Part 2 of 4

Skill	ONS Survey Grouping	Our Grouping
Physical stamina (to work for long periods on physical activities)	Technical / practical	Manual
Skill or accuracy in using hands (for example, to mend, repair, assemble, construct or adjust things)	Technical / practical	Manual
Knowledge of particular products or services	Technical / practical	Cognitive
Specialist knowledge or understanding	Technical / practical	Cognitive
Reading and understanding instructions, guidelines, manuals or reports	Technical / practical	Cognitive
Writing instructions, guidelines, manuals or reports	Technical / practical	Cognitive
Number skills	Technical / practical	Cognitive
Measuring, calculating or estimating	Technical / practical	Cognitive
Spotting problems or faults	Technical / practical	Cognitive
Thinking of solutions to problems	Technical / practical	Cognitive
Analysing complex problems in detail	Technical / practical	Cognitive
Checking things to ensure there are no errors	Technical / practical	Cognitive
Following instructions precisely	Technical / practical	Cognitive

Table A19: Applicant Skills - Part 3 of 4

Skill	**ONS Survey Grouping**	**Our Grouping**
Adapting to new equipment or materials	Technical / practical	Cognitive
Using or operating tools or equipment	Technical / practical	Manual
Computer literacy	IT	Cognitive
Using a computer for word processing, email etc	IT	Cognitive
Using computer applications for carrying out specialised tasks	IT	Cognitive
Writing computer software	IT	Cognitive
Designing, building, repairing computer hardware	IT	Cognitive
Building or repairing electronic equipment	Technical / practical	Cognitive
Knowledge of how different materials behave	Technical / practical	Cognitive
Knowledge of production processes	Technical / practical	Cognitive
Technical drawing	Technical / practical	Cognitive
Understanding diagrams, drawings or blueprints	Technical / practical	Cognitive
Artistic or creative skills	Technical / practical	Dropped from data

Table A20: Applicant Skills - Part 4 of 4

Skill	**ONS Survey Grouping**	**Our Grouping**
Design skills	Technical / practical	Cognitive
Foreign language skills	Technical / practical	Cognitive
Driving or operating vehicles	Technical / practical	Manual
Knowledge of safety issues	Technical / practical	Cognitive
Knowledge of relevant law or regulations	Technical / practical	Cognitive
Keeping records	Technical / practical	Cognitive
Researching facts or information	Technical / practical	Cognitive
Using statistics	Technical / practical	Cognitive
Organising time or activities	Technical / practical	Dropped from data
Being able to work at speed	Technical / practical	Dropped from data
Working accurately under pressure	Technical / practical	Dropped from data
Manual skills	Technical / practical	Manual
Observing, inspecting, detecting	Technical / practical	Cognitive
Knowledge of organisation and planning	Technical / practical	Cognitive
Performing or entertaining	Technical / practical	Dropped from data