

Online Appendix

A Details of computation

In the model, a worker is characterized by (i) her labor market state: employed (with a job), unemployed (not employed but actively searching for a job), not in the labor force (not employed and not searching for a job), (ii) her wealth, a , (iii) her idiosyncratic general productivity, z , (iv) her match-specific productivity (if employed), μ , (v) her match-specific productivity in the previous job (if unemployed and eligible for UI), and (vi) her age, j . Age in the model is monthly and ranges from 1 to 947. Model age $j = 1$ corresponds to (annual) age 22 in the data. At each age j , an s_j fraction of workers survives to age $j + 1$. Everyone dies for sure at age 947, which corresponds to one month before age 101 in the data.

To solve the model numerically, we discretize the state space. For assets, a , we create a log-spaced grid of 100 points between 0 and 380. The discretization of asset space is independent of other model parameters. We also discretize the state space of z and μ . However, the discretization of z -space and μ -space depend on other model parameters and are explained later.

Our model is in general equilibrium. We assume competitive markets, and thus (i) the marginal product of capital minus depreciation rate must be equal to the interest rate in the household problem, and (ii) the marginal product of labor must be equal to the wage in efficiency units. Moreover, the government budget constraint must hold: total labor tax collection plus the assets of deceased workers must be equal to total transfer payments and total UI payments. Since we normalize wage in the efficiency unit to 1, we achieve condition (ii) by setting A in the production function to the value that ensures the marginal product of labor is equal to 1.

Typically, one would use a standard nested algorithm in which the inner loop calculates the general equilibrium for each set of parameter values, and the outer loop minimizes the distance between the data and model moments. Due to computational complexity, we solve the minimization and general equilibrium together by adding the capital market equilibrium condition and government budget constraint as additional targets to be minimized. This procedure speeds up the calibration process substantially.

Recall that $\xi \equiv \{\beta, \delta, \lambda_{e,2}, \lambda_{e,1}, \lambda_{e,0}, \lambda_{u,2}, \lambda_{u,1}, \lambda_{u,0}, \lambda_{n,2}, \lambda_{n,1}, \lambda_{n,0}, \sigma_2, \sigma_1, \sigma_0, g_2, g_1, g_0, \psi, \gamma, \sigma_\mu, \sigma_z, h, \zeta, \alpha, b_0, \bar{b}\}$ is the set of parameters to be calibrated within the model. Let $\xi^o \equiv \{\{s_j\}_{j=1}^{947}, \rho_\mu, \rho_z\}$ be the set of parameters calibrated externally.

A.1 Solution steps

We solve our model as follows:

1. Given parameter values ξ and ξ^o , we first create the discrete state space to solve the model numerically.

- (a) We discretize the AR(1) (log-) idiosyncratic productivity process using the Tauchen method. We create a grid of idiosyncratic productivity, z -grid, consisting of 15 points. The Tauchen method also generates transition probabilities from current idiosyncratic productivity, z , to the next period's idiosyncratic productivity, z' : $P_{z,z'}$ for z and z' in z -grid.
- (b) Similarly, we discretize the AR(1) (log-) match-specific productivity process of the worker-firm pair using the Tauchen method. μ -grid consists of 15 points. The probability of match-specific productivity, μ , becoming μ' in the next period if the worker-firm pair survives is denoted as $P_{\mu,\mu'}$ for μ and μ' in μ -grid.
- (c) Match-specific productivity for a new job (for the workers in the leisure islands and for the workers who have a job but receive an outside offer) is drawn from a Pareto distribution:

$$\Pr[M > \mu] = \begin{cases} \left(\frac{\mu_1}{\mu}\right)^\alpha & \text{for } \mu \geq \mu_1, \\ 1 & \text{for } \mu < \mu_1, \end{cases}$$

where μ_1 is the lowest point in the μ -grid. Note that M is the random variable, and μ is its realization. Let μ_k be the k -th lowest point in the μ -grid. Then, the probability of receiving an outside offer with match-specific productivity of μ_k is equal to:

$$S(\mu_k) = \begin{cases} \left(\frac{\mu_1}{\mu_{k-1}}\right)^\alpha - \left(\frac{\mu_1}{\mu_k}\right)^\alpha & \text{for } k > 1, \\ 0 & \text{for } k = 1. \end{cases}$$

- (d) Recall that with a probability, ζ , match-specific productivity is unrevealed (or unknown). To account for the unknown state, we add one more grid point to μ -grid, which now consists of 16 points. We assume that if the match-specific productivity is not known, workers are paid as if they have the median match-specific productivity. Let $P_{\mu,\mu'}^{ext}$ represent transition probabilities in the extended μ grid:

$$P_{\mu,\mu'}^{ext} = \begin{cases} P_{\mu,\mu'} & \text{if both } \mu \text{ and } \mu' \text{ are known,} \\ 0 & \text{if } \mu \text{ is known but } \mu' \text{ is unknown,} \\ (1 - \zeta)S(\mu') & \text{if } \mu \text{ is unknown but } \mu' \text{ is known,} \\ \zeta & \text{if both } \mu \text{ and } \mu' \text{ are unknown.} \end{cases} \quad (1)$$

Similarly, let $S^{ext}(\mu_k)$ be the probability of receiving an outside job offer with match-specific productivity, μ_k , while taking into account that match quality might be unknown.

$$S^{ext}(\mu_k) = \begin{cases} (1 - \zeta)S(\mu_k) & \text{if } \mu_k \text{ is known,} \\ \zeta & \text{if } \mu_k \text{ is unknown.} \end{cases}$$

2. Given parameter values ξ and ξ^o , a guess for the interest rate r , normalized wage rate, $\tilde{\omega} = 1$, and a guess for the government transfer to households, \mathbf{T} , we recursively solve for the value functions of the workers: $W_j(a, z, \mu)$, $U_j(a, z)$, $\tilde{U}_j(a, z, \mu)$, $N_j(a, z)$, $T_j(a, z, \mu)$, $S_j(a, z, \mu)$, $O_j(a, z)$, $\tilde{O}_j(a, z, \mu)$, $F_j(a, z)$, and $\tilde{F}_j(a, z, \mu)$. Starting from age $j = 947$, when the continuation value is equal to 0, we solve for the consumption/saving decision of age $j = 947$ workers and calculate the value function as described in Section 3.2. We iterate this process until we reach age $j = 1$. We linearly interpolate the continuation value and solve for the optimal saving decision using the golden section search method.
3. Using the value functions from step 2, we generate the decision rules for labor force participation, accepting/rejecting an offer, and switching jobs, as described in Section 3.5.
4. Starting from age $j = 1$, we simulate the model using the decision rules from step 3. We assume all the workers are born at the leisure island with no assets and no wage offer at hand. A newborn worker's initial idiosyncratic productivity is drawn from the long-run idiosyncratic productivity distribution.
5. After observing gross worker flows between three labor market states and job-to-job flows, we calculate the transition rates as follows (EU used as an example):

$$EU^{model}(\xi, j) = \frac{\text{Measure of the employed at age } j \text{ moving to unemployment the next period}}{\text{Measure of the employed at age } j}.$$

Because our model is stationary and no aggregate shocks occur, we drop the time index. We calculate age-specific transition rates between labor market states by taking the mean of the monthly transition rates. Continuing with the EU transition as an example,

$$EU^{model}(\xi, j^a) = \frac{1}{12} \sum_{j=12(j^a-22)+1}^{12(j^a-21)} EU^{model}(\xi, j),$$

where j^a is the (annual) age. We assume model-age $j = 1$ corresponds to age 22 in the data. We calculate the monthly wage rate by taking the average of wages of same-aged workers. Then, we convert the monthly wage rate to the age-specific wage as above. However, we normalize the average wage at age 42 to 1:

$$\bar{\omega}(\boldsymbol{\xi}, j^a) = \frac{1}{12} \sum_{j=12(j^a-22)+1}^{12(j^a-21)} \omega^{model}(\boldsymbol{\xi}, j),$$

$$\omega^{model}(\boldsymbol{\xi}, j^a) = \frac{\bar{\omega}(\boldsymbol{\xi}, j^a)}{\bar{\omega}(\boldsymbol{\xi}, 42)}.$$

6. We solve for a stationary equilibrium in which the interest rate and wage rate are constant and the distribution of agents over the state space is stationary. Hence, given the model parameters, interest rate, and wage rate, total capital supply in the economy is equal to $K = \int a_i di$ and total labor supply in efficiency units is equal to $\int e(i) z_i \mu_i g_i di = L$, where i represents a worker and $e(i)$ is the employment status of the workers with $e(i) = 1$ if i is employed and 0 otherwise. The integration is over all the workers (both in the work island and leisure island) in the model.
7. The next step is to ensure general equilibrium labor-market-clearing condition. Having solved for K and L in step 6, we choose A such that $(1 - \theta)A(K/L)^\theta = 1$ to set the marginal product of labor to 1 in equilibrium.
8. We calculate i) the log difference between the interest rate and marginal product of capital minus depreciation, $\log(r) - \log(\theta A(K/L)^{\theta-1} - \delta)$, and ii) the log difference between tax collection plus assets of the deceased workers and transfer payments.
9. We repeat steps 2 to 8 with new r and T values until the sum of the square of the two log differences from step 8 gets close to 0.

A.2 Calibration procedure

To calibrate the model, we use the following algorithm.

1. Set $\boldsymbol{\xi}^o$ to their respected values as described in Section 4.
2. For an initial $\boldsymbol{\xi}$, \mathbf{T} , and r , we solve and simulate the model from steps 2 to 8 and calculate the gross worker flows from the simulated data. Instead of doing step 9 of the model solution algorithm in every iteration, we combine step 9 of the model solution algorithm with the calibration algorithm as described in the following points.
3. Let $X^{model}(\boldsymbol{\xi})$ be the collection of transition rates among worker states, normalized average wages and labor market ratios for the workers at (annual) ages between 23 and 70,¹ the interest rate, the investment-to-GDP ratio (x), the total-UI-payments-to-total-earnings ratio (y), and the UI-cap-to-average-wage ratio (m):

¹Recall that our model starts with annual age of 22, but we only consider ages between 23 and 70 in the calibration.

$$\begin{aligned}
X^{model}(\boldsymbol{\xi}) = & \text{vec} \left(\left\{ EU^{model}(\boldsymbol{\xi}, j^a), EN^{model}(\boldsymbol{\xi}, j^a), EE^{model}(\boldsymbol{\xi}, j^a), \right. \right. \\
& NU^{model}(\boldsymbol{\xi}, j^a), NE^{model}(\boldsymbol{\xi}, j^a), UN^{model}(\boldsymbol{\xi}, j^a), \\
& UE^{model}(\boldsymbol{\xi}, j^a), \omega^{model}(\boldsymbol{\xi}, j^a), \\
& UR^{model}(\boldsymbol{\xi}, j^a), LFPR^{model}(\boldsymbol{\xi}, j^a), E/pop^{model}(\boldsymbol{\xi}, j^a) \left. \right\}_{j^a=23}^{70}, \\
& r(\boldsymbol{\xi}), x(\boldsymbol{\xi}), y(\boldsymbol{\xi}), m(\boldsymbol{\xi}) \Big),
\end{aligned}$$

and let X^{data} be the collection of the six-year rolling average of transition rates, normalized average wages, the labor market ratios² of males at ages between 23 and 70 observed in the data, the interest rate target, and the investment-to-GDP ratio target, the UI-payments-to-total-earnings target, and the UI-cap-to-average-wage target:

$$\begin{aligned}
X^{data} = & \text{vec} \left(\left\{ EU^{data}(j^a), EN^{data}(j^a), EE^{data}(j^a), \right. \right. \\
& NU^{data}(j^a), NE^{data}(j^a), UN^{data}(j^a), \\
& UE^{data}(j^a), \omega^{data}(j^a), \\
& UR^{data}(j^a), LFPR^{data}(j^a), E/pop^{data}(j^a) \left. \right\}_{j^a=23}^{70}, r, x, y, m \Big).
\end{aligned}$$

We characterize the loss function as follows:

$$\mathcal{L} \equiv |(\log X^{model}(\boldsymbol{\xi}) - \log X^{data})'W|(\log X^{model}(\boldsymbol{\xi}) - \log X^{data})|,$$

where W is a diagonal weighting matrix. We set all but the last four elements of the diagonal of W to be equal to 1, and the last four elements to be equal to 100.³

We modify the loss function, \mathcal{L} , by adding the capital market equilibrium condition and the government budget condition to \mathcal{L} . We define the modified loss function, $\tilde{\mathcal{L}}$, that we are minimizing as

$$\begin{aligned}
\tilde{\mathcal{L}} = & \mathcal{L} + \tilde{W} \left[\log(\theta A(K/L)^{\theta-1} - \delta) - \log(r(\boldsymbol{\xi})) \right. \\
& + \log \left(\mathbf{T} + \int b(i)di \right) \\
& \left. - \log \left(\tau \int e(i)\omega_{j(i)}(i)(\mu(i), z(i))di + \int a(i)(1 - s(i))di + \tau \int b(i)di \right) \right],
\end{aligned}$$

²The average wage at each age is expressed relative to the average wage of 42-year-old male workers.

³We chose 100 as a moderately large number to give aggregate variables higher weights than a flow rate at a particular age in our minimization algorithm.

where $e(i)$ is the employment status of agent i with 1 for employed and 0 for not employed, $s(i)$ is the survival status of agent i , 1 for surviving agents, and 0 for deceased agents, $b(i)$ is the UI payment of individual i , and \tilde{W} is a large number to ensure that the market-clearing condition and the government budget constraint are always satisfied at optimum.⁴

4. We repeat steps 2 and 3 for different ξ , \mathbf{T} and r values until the modified loss function is minimized. We use the Powell method from Scipy minimization sub-package as our minimizer.

⁴In the solution we set \tilde{W} equal to 38400, a large number to ensure equilibrium conditions are always satisfied in our minimization algorithm. Remember that we combine equilibrium conditions and distance between data and model moments in a single loop in our minimization algorithm to keep the running time of the code at manageable levels. Separating the two would increase already very long running time to unmanageable levels.

B Robustness checks with ρ_z and ρ_μ

We check the robustness of our results with the level of persistence in match-specific productivity and idiosyncratic productivity. Remember that in our benchmark calibration we set $\rho_z = 0.97$ and $\rho_\mu = 0.98$. We try two additional persistence levels: i) $\rho_z = 0.98$ and $\rho_\mu = 0.98$ and ii) $\rho_z = 0.98$ and $\rho_\mu = 0.97$. In each case, we keep our remaining benchmark calibration parameters but find the interest rate, r , which clears the capital market, A which clears the labor market and normalizes $\tilde{\omega}$ to 1, and the transfer, \mathbf{T} , which clears the government budget constraint. Figure 1 shows the labor market ratios and Figure 2 shows the gross labor flows with different persistence parameters. Despite not calibrating the model fully, we observe that labor market ratios and gross worker flow rates are similar across models with differing productivity persistence parameters.

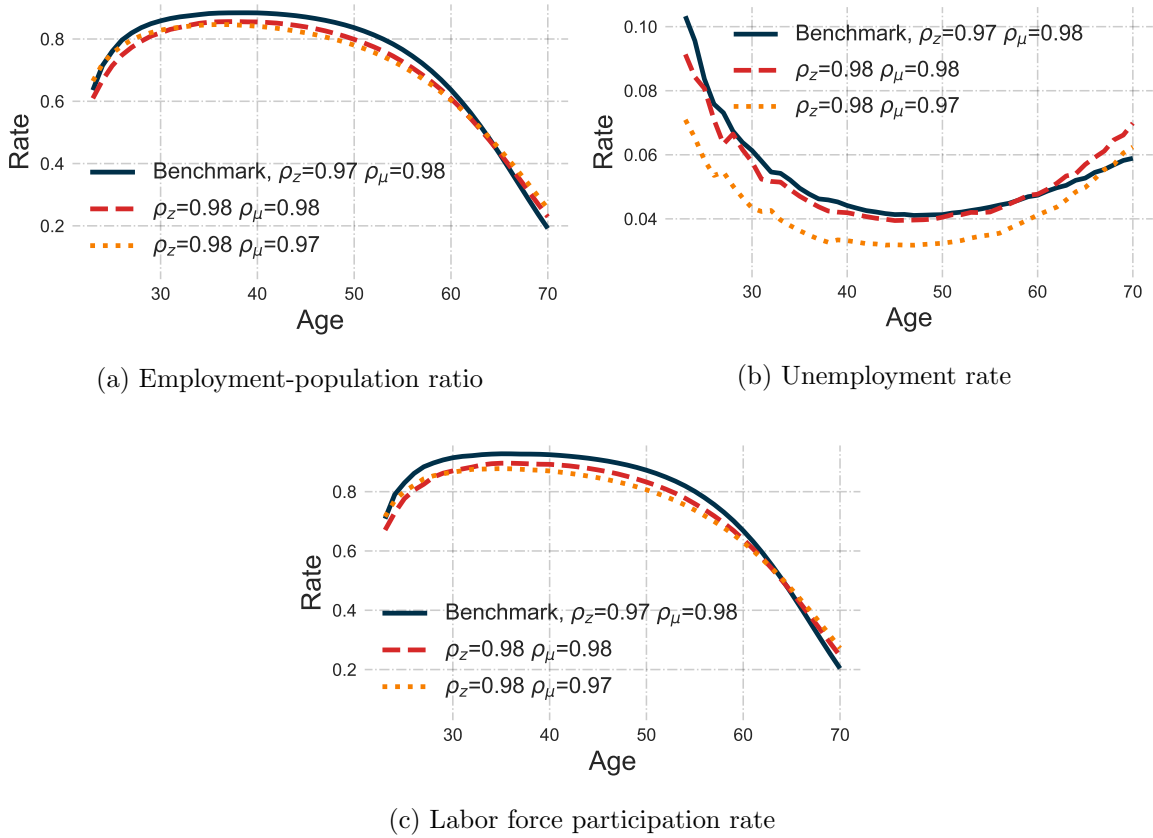


Figure 1: Labor market ratios after with different persistence parameters

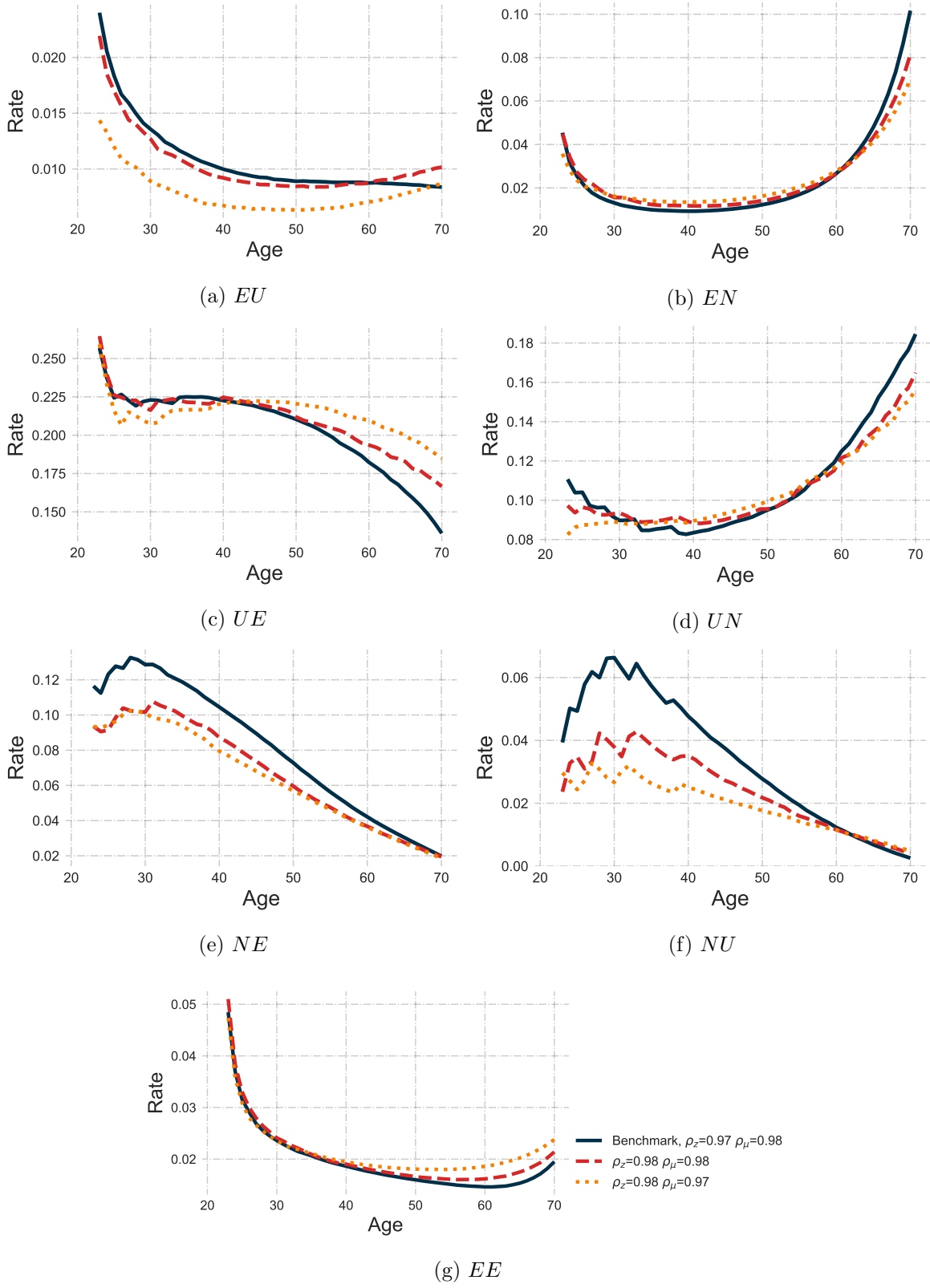


Figure 2: Gross worker flow rates with different persistence parameters

C Calibration results for various specifications for $\lambda_e(j)$, $\lambda_u(j)$, $\lambda_n(j)$, $\sigma(j)$, and $g(j)$

This Appendix presents the calibration results for Cases 1 to 4 in Section 4. Table 1 presents the resulting parameter values. The “Case 4” column repeats the numbers in Table 1 in the main text.

Figure 3 plots the estimated λ_e , λ_n , λ_u , σ , and g as functions of age. In all cases, estimated λ_e , λ_u , and λ_n are relatively flat over the age. Figure 4 plots the stocks in each experiment. Figure 5 draws the corresponding flows and wages over the life cycle.

In each figure, “Case 1” in the main text corresponds to the label “age-independent.” For stocks, this specification cannot account for the steep decline of the employment-population ratio and the labor force participation rate in old age. The unemployment rate is almost flat over the life cycle, which is at odds with the significantly larger unemployment rate for young workers in the data.

On the flow rates, the model outcome is qualitatively consistent with the data for some of the flows. For EE , UE , and NE flows, the data exhibits declining patterns over age. These three flows are related to job-finding, and the success reflects the fact that the estimated outcome with flexible parameters exhibits relatively flat profiles for λ_e , λ_u , and λ_n . Similarly, for the NU flow, both data and model profiles are monotonically decreasing over age. In contrast, the model outcome is flat for the EU flow. The model flow fails to replicate the U-shape pattern in the data for EN and UN . These flows reflect the labor supply decision, and for the labor supply decision, age-dependent productivity is an important driver for the flows.

“Case 2” in the main text corresponds to “age-dependent prod” in the figures. There, the productivity profile is allowed to be age-dependent, and it is estimated to have an inverted-U shape. The stocks in Figure 4 show that the model fit to the stock is quite good. The flows in 5 also achieve a very good fit to the data.

“Case 3” corresponds to “age-dependent job offer rates” in the figures. The outcome turned out to be very similar to the Case 1. Allowing the job-finding rates to be age-dependent does not contribute significantly to the match of stocks and flows to the data.

“Case 4” is denoted as “age-dependent.” These are identical to the model outcomes presented in the main text.

Notation	Definition	Case 1	Case 2	Case 3	Case 4
β	Discount factor	0.997	0.997	0.997	0.997
θ	Elasticity of output w.r.t. capital	0.300	0.300	0.300	0.300
δ	Depreciation rate	0.009	0.009	0.009	0.009
ψ	Disutility of active job search	0.063	0.062	0.064	0.058
A	Total factor productivity	0.489	0.488	0.490	0.489
ρ_μ	Persistence parameter of monthly AR(1) match-specific productivity	0.980	0.980	0.980	0.980
σ_μ	Std. dev. of innovations in match-specific productivity	0.112	0.108	0.112	0.107
ρ_z	Persistence parameter of monthly AR(1) idiosyncratic productivity	0.970	0.970	0.970	0.970
σ_z	Std. dev. of innovations in idiosyncratic productivity	0.157	0.100	0.143	0.093
h	Home productivity	0	0.136	0	0.120
ζ	Unknown match quality probability	0.151	0.241	0.336	0.275
α	Shape parameter of Pareto distribution	5.455	7.797	3.375	6.934
$\bar{\mu}$	Match quality for unrevealed matches	1	1	1	1
γ	Disutility of work over disutility of active job search	8.764	8.873	9.236	8.850
\mathbf{T}	Transfer	0.195	0.226	0.205	0.207
b_0	UI replacement rate	0.309	0.584	0.351	0.419
\bar{b}	UI payment cap	0.388	0.558	0.431	0.498
χ	Initial UI takeup rate	0.770	0.770	0.770	0.770
η	The probability of losing UI benefits	0.167	0.167	0.167	0.167
J	Monthly age at which everyone dies	947	947	947	947

Table 1: Age-independent parameters

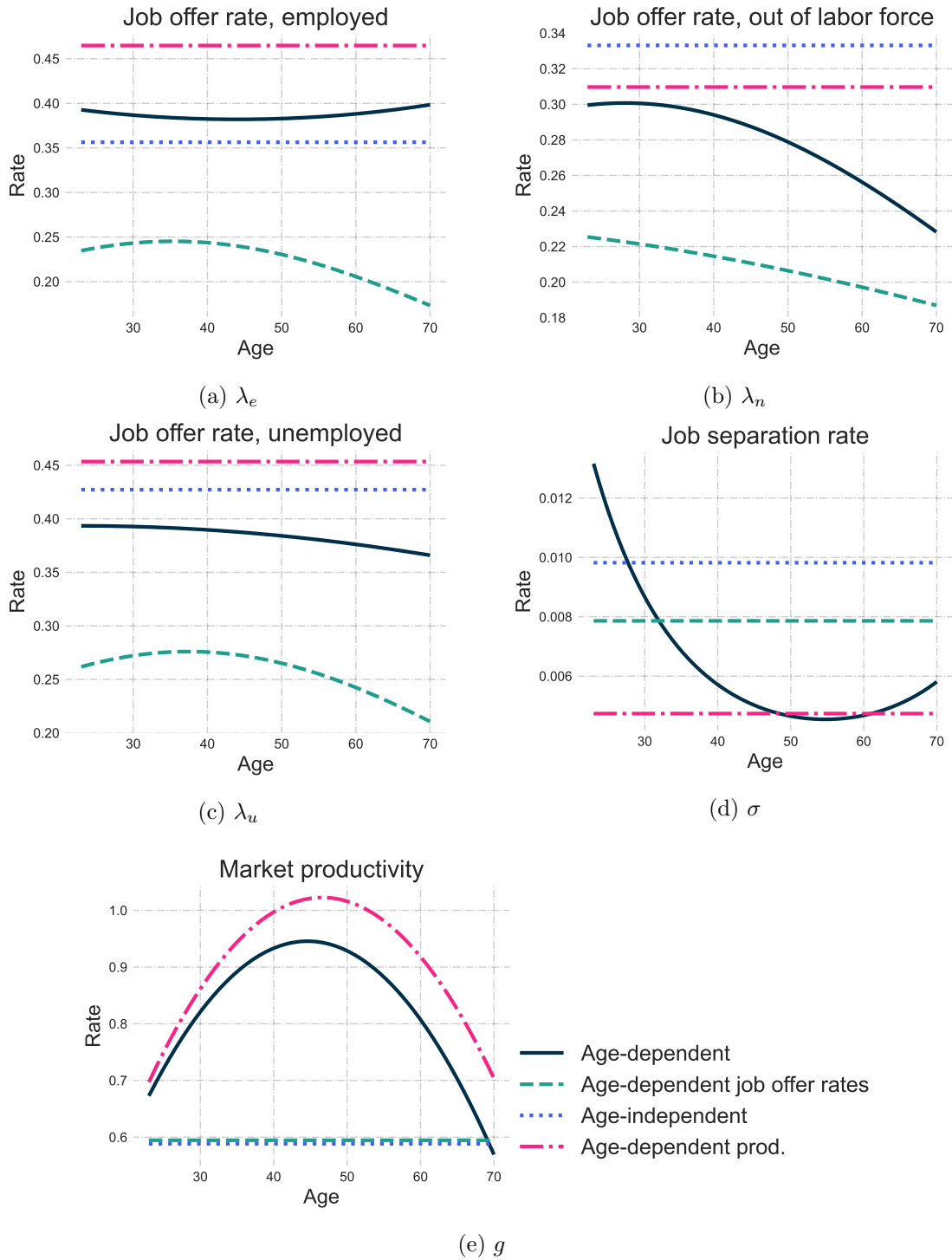
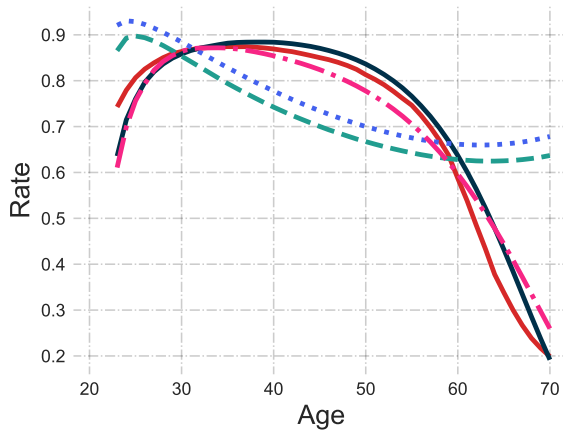
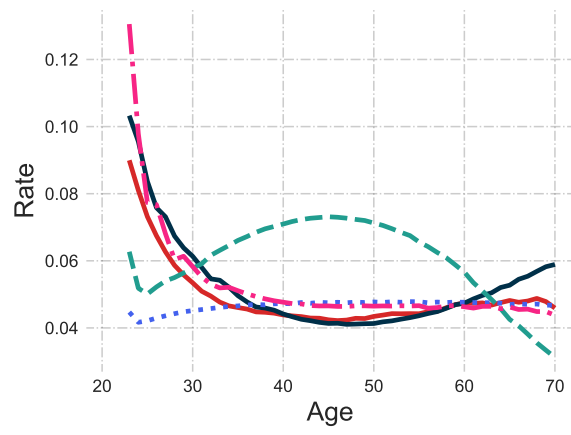


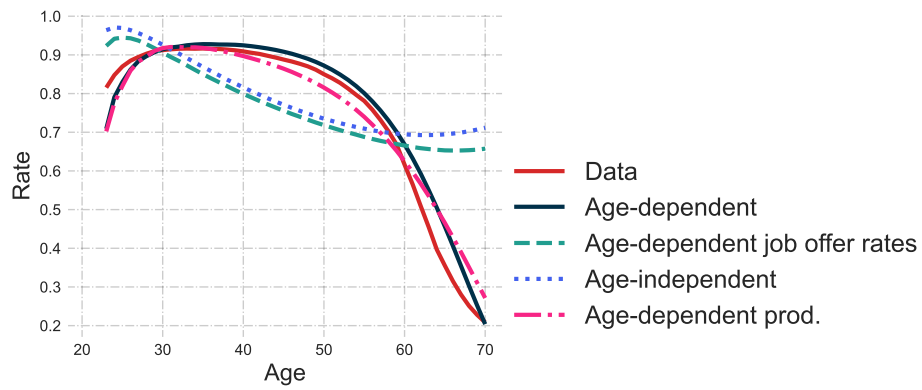
Figure 3: Age-dependent parameters



(a) Employment-population ratio

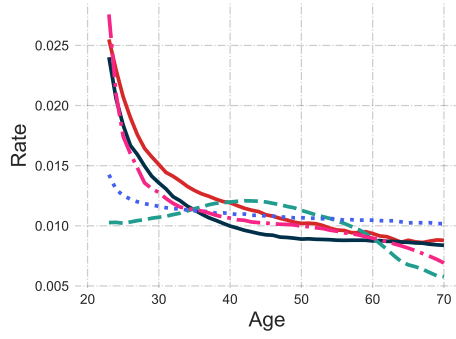


(b) Unemployment Rate

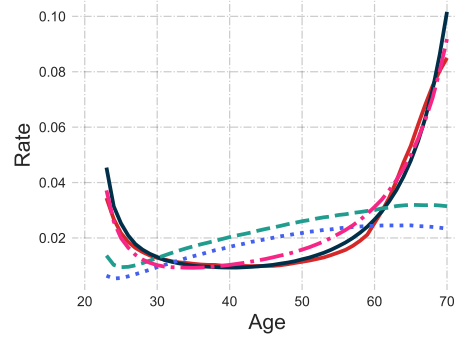


(c) Labor Force Participation Rate

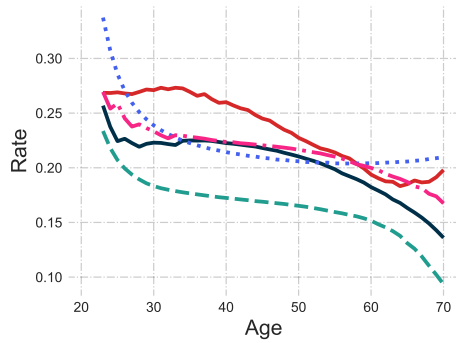
Figure 4: Stocks for each cases



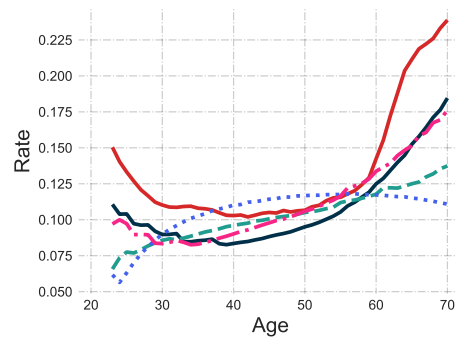
(a) *EU*



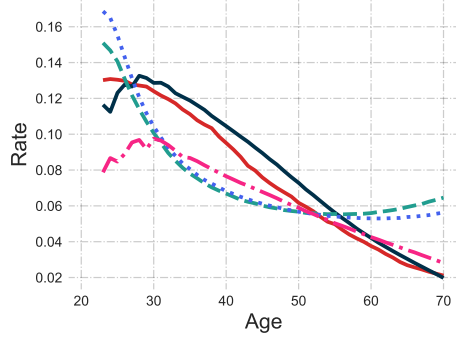
(b) *EN*



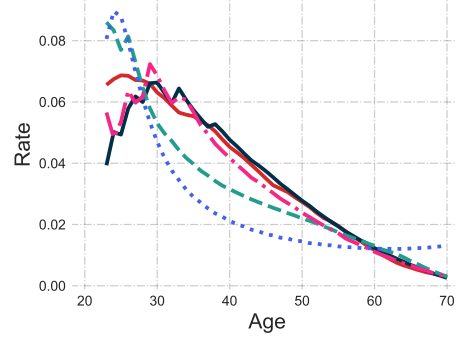
(c) *UE*



(d) *UN*



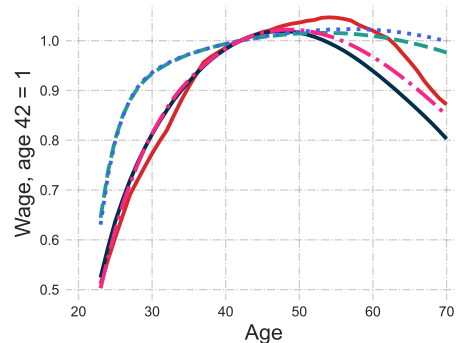
(e) *NE*



(f) *NU*



(g) *EE*



(h) wages

Figure 5: Flows and wages for each cases

D Age coefficients on frictions and productivity: baseline

The age component of market productivity, $g(j)$, the logarithms of job-offer arrival rates, $\log \lambda_e(j)$, $\log \lambda_u(j)$, $\log \lambda_n(j)$, and the logarithm of exogenous job-separation rate are characterized as second-degree polynomials of age, j :

$$\begin{aligned}\lambda_e(j) &= \exp(\lambda_{e,2}j^2 + \lambda_{e,1}j + \lambda_{e,0}), \\ \lambda_u(j) &= \exp(\lambda_{u,2}j^2 + \lambda_{u,1}j + \lambda_{u,0}), \\ \lambda_n(j) &= \exp(\lambda_{n,2}j^2 + \lambda_{n,1}j + \lambda_{n,0}), \\ \sigma(j) &= \exp(\sigma_2j^2 + \sigma_1j + \sigma_0), \\ g(j) &= g_2j^2 + g_1j + g_0.\end{aligned}$$

The calibrated coefficients are shown in Table 2.

Table 2: Coefficients of polynomials

Parameter	Definition	Case 1	Case 2	Case 3	Case 4
$\lambda_{u,2}$	Log of the job offer rate of the unemployed	0	0	-1.774226e-06	-2.269305e-07
$\lambda_{u,1}$		0	0	6.592503e-04	4.987631e-06
$\lambda_{u,0}$		-0.850382	-0.790835	-1.348507	-9.327986e-01
$\lambda_{e,2}$	Log of the job offer rate of the employed	0	0	-2.017359e-06	4.339772e-07
$\lambda_{e,1}$		0	0	6.486581e-04	-2.299751e-04
$\lambda_{e,0}$		-1.031886	-0.766102	-1.457477	-9.317682e-01
$\lambda_{n,2}$	Log of the job offer rate of the nonparticipant	0	0	-2.525953e-07	-1.092234e-06
$\lambda_{n,1}$		0	0	-1.835180e-04	1.601267e-04
$\lambda_{n,0}$		-1.099198	-1.172103	-1.487424	-1.207363
σ_2	Log of the job separation rate	0	0	0	7.361115e-06
σ_1		0	0	0	-5.787838e-03
σ_0		-4.623172	-5.352062	-4.845629	-4.256557
g_2	Age component of market productivity	0	-0.000004	0	-4.069923e-06
g_1		0	0.002407	0	2.213185e-03
g_0		0.588203	0.666449	5.945531e-01	6.447473e-01

E Wealth

This section compares the wealth-income ratio distribution in the model with the data. We rely on [Kuhn and Ríos-Rull \(2016\)](#) for wealth statistics in the US, calculated from the Survey of Consumer Finances (SCF). We normalize the average wealth and the average income in each category and year by the average wealth and income in the entire economy in the year. Figure 6 shows the wealth-income ratio in each category averaged over the years (bars) and the confidence intervals (lines on bars).

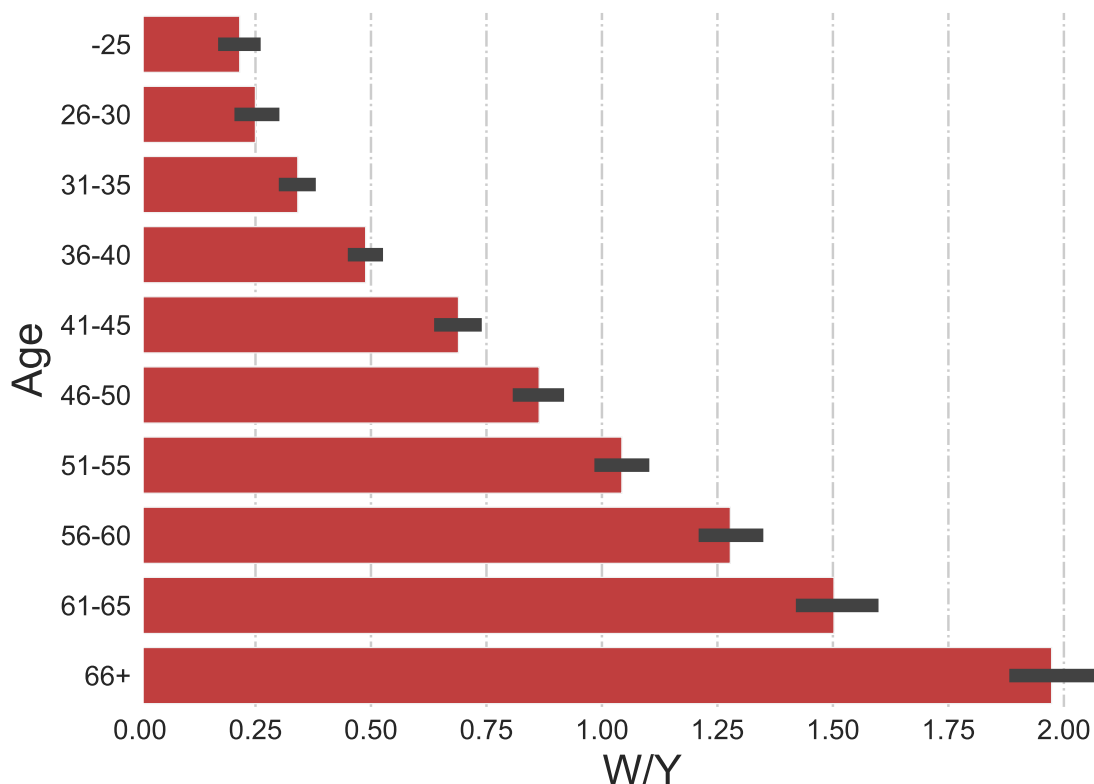


Figure 6: Average wealth-income ratio from data

Data from [Kuhn and Ríos-Rull \(2016\)](#). Accessed from <https://sites.google.com/site/kuhnecon/home/us-inequality>. The values are averages of wealth-income ratio over years.

Table 3 shows the wealth-income ratio in each category of age and labor market state. It also presents the results for the counterfactual experiments.

Benchmark			
Age	Employed	Nonparticipant	Unemployed
23 -25	0.06	0.04	0.10
26-30	0.19	0.43	0.34
31-35	0.37	1.80	0.74
36-40	0.54	3.36	1.19
41-45	0.71	4.65	1.65
46-50	0.87	5.58	2.06
51-55	1.00	6.10	2.32
56-60	1.06	6.12	2.36
61-65	1.03	5.62	2.14
66+	0.85	2.56	1.57
60% Replacement Rate			
Age	Employed	Nonparticipant	Unemployed
23-25	0.06	0.04	0.09
26-30	0.19	0.43	0.31
31-35	0.36	1.79	0.67
36-40	0.54	3.35	1.09
41-45	0.71	4.65	1.52
46-50	0.87	5.58	1.92
51-55	1.00	6.09	2.16
56-60	1.06	6.12	2.20
61-65	1.03	5.62	1.99
66+	0.85	2.56	1.44
45% Tax			
Age	Employed	Nonparticipant	Unemployed
23-25	0.07	0.04	0.12
26-30	0.20	0.23	0.34
31-35	0.35	0.95	0.65
36-40	0.51	2.15	1.03
41-45	0.67	3.43	1.42
46-50	0.81	4.43	1.73
51-55	0.90	4.90	1.90
56-60	0.90	4.83	1.84
61-65	0.82	4.28	1.56
66+	0.63	1.73	1.13

Table 3: Average wealth/income in age group-worker state pairs.

E.1 Taxes and transfers

Table 3 shows the wealth-income ratio in each category of age and labor market state. It also presents the results of the counterfactual experiments.

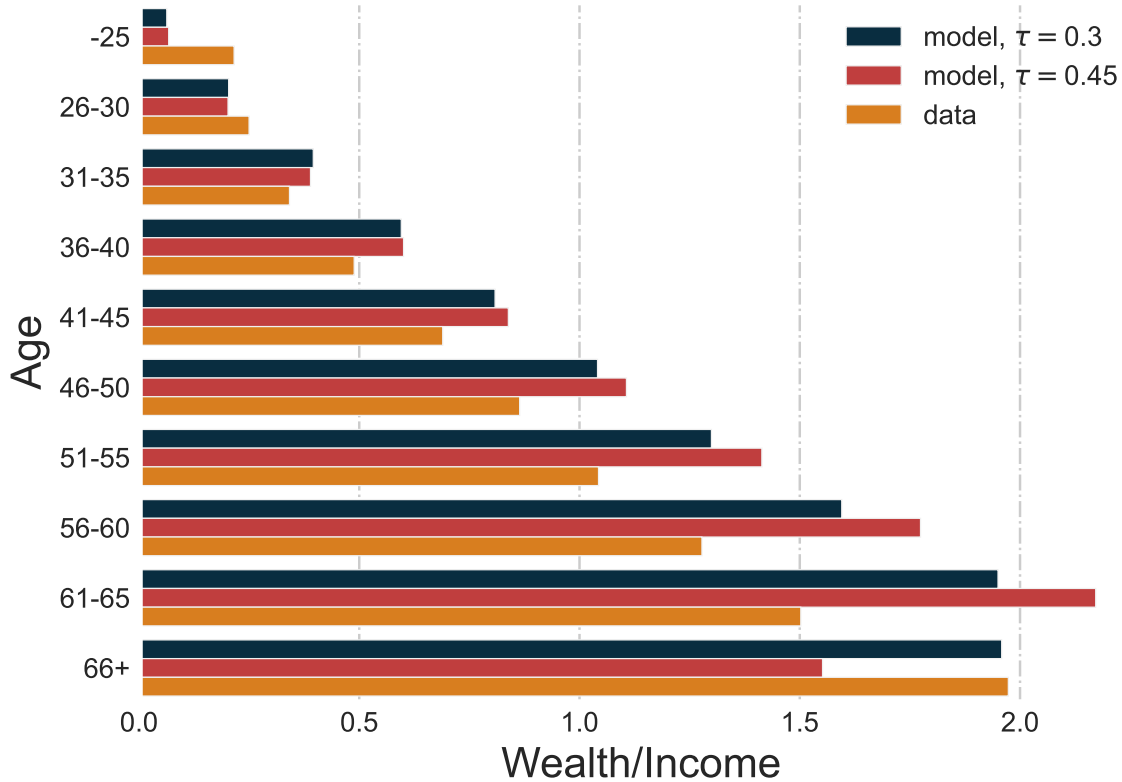


Figure 7: Average wealth-income ratio from model and data.

Data: averages across years from Kuhn and Ríos-Rull (2016). Accessed from <https://sites.google.com/site/kuhnecon/home/us-inequality>. Wealth and income are relative to the average wealth and average income in the entire economy.

E.2 UI experiment

Figure 8 compares the average wealth-income ratio in the model and UI (60% replacement rate) with data. Somewhat surprisingly, the response of wealth distribution to the change in UI is very small.

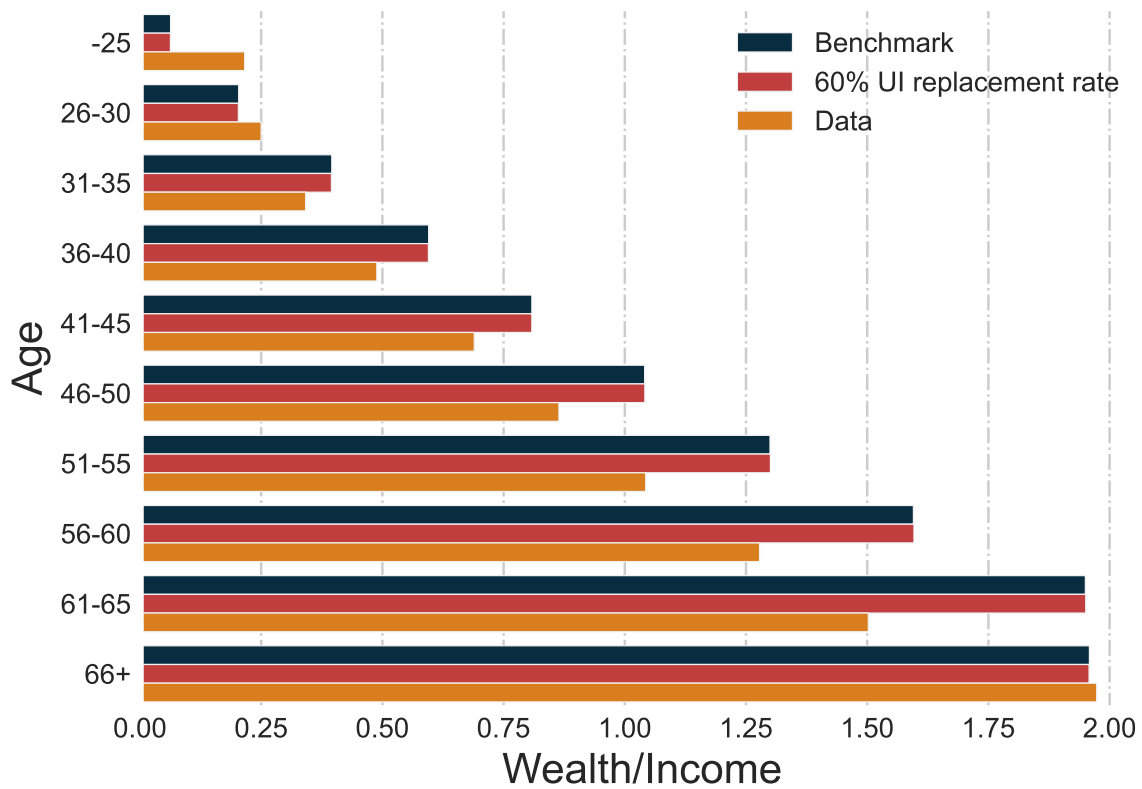


Figure 8: Average wealth from model and data

Data: averages across years from [Kuhn and Ríos-Rull \(2016\)](https://sites.google.com/site/kuhnecon/home/us-inequality). Accessed from <https://sites.google.com/site/kuhnecon/home/us-inequality>. Wealth and income are relative to the average wealth and average income in the entire economy.

F Welfare Gain

Recall that we define the utility as:

$$\mathbf{U}_\omega = \sum_{j=1}^J \left(\beta^j \prod_{t=1}^j s_t \right) E_0[\log c_j - d_j].$$

Here, $\omega \in \{U, N\}$ is the initial labor market state (a newborn starts from nonemployment).

We calculate the welfare gain as follows:

1. Solve for the indirect utility of a newborn with zero wealth and idiosyncratic productivity z when she is unemployed and out of the labor force in the benchmark economy: $\mathbf{U}_U(a = 0, z, \tau = 0.3)$ and $\mathbf{U}_N(a = 0, z, \tau = 0.3)$

2. Similarly, solve for the indirect utility when the tax rate is 45 percent: $\mathbf{U}_U(a = 0, z, \tau = 0.45)$ and $\mathbf{U}_N(a = 0, z, \tau = 0.45)$

3. Find the extra consumption (in every period) a worker in the benchmark economy at age 1 with 0 assets and idiosyncratic productivity z asks to make him indifferent between being in the benchmark economy and being in the 45-percent-tax economy. Call this function $\xi_\omega(z)$:

$$\sum_{j=1}^J \left(\beta^j \prod_{t=1}^j s_t \right) E_0[\log \xi_\omega(z) c_j^b - d_j^b] = \sum_{j=1}^J \left(\beta^j \prod_{t=1}^j s_t \right) E_0[\log c_j^{45} - d_j^{45}],$$

where superscripts b and 45 represent benchmark and 45-percent-tax economy.

Then,

$$\sum_{j=1}^J \left(\beta^j \prod_{t=1}^j s_t \right) \log \xi_\omega(z) + \sum_{j=1}^J \left(\beta^j \prod_{t=1}^j s_t \right) E_0[\log c_j^b - d_j^b] = \sum_{j=1}^J \left(\beta^j \prod_{t=1}^j s_t \right) E_0[\log c_j^{45} - d_j^{45}]$$

Then substitute the indirect utilities. For example, when $\omega = U$,

$$\sum_{j=1}^J \left(\beta^j \prod_{t=1}^j s_t \right) \log \xi_U(z) + \mathbf{U}_U(a = 0, z, \tau = 0.3) = \mathbf{U}_U(a = 0, z, \tau = 0.45)$$

Then

$$\log \xi_U(z) = \frac{\mathbf{U}_U(a = 0, z, \tau = 0.45) - \mathbf{U}_U(a = 0, z, \tau = 0.3)}{\sum_{j=1}^J \left(\beta^j \prod_{t=1}^j s_t \right)}$$

4. Then calculate the expected welfare gain of a newborn with 0 wealth before her idiosyncratic productivity is revealed, and she makes a decision to participate in the labor force:

$$\tilde{\xi} \equiv \sum_{z=1}^Z S_z \left((\xi_U(z) - 1) 1_p(a = 0, z) + (\xi_N(z) - 1) (1 - 1_p(a = 0, z)) \right),$$

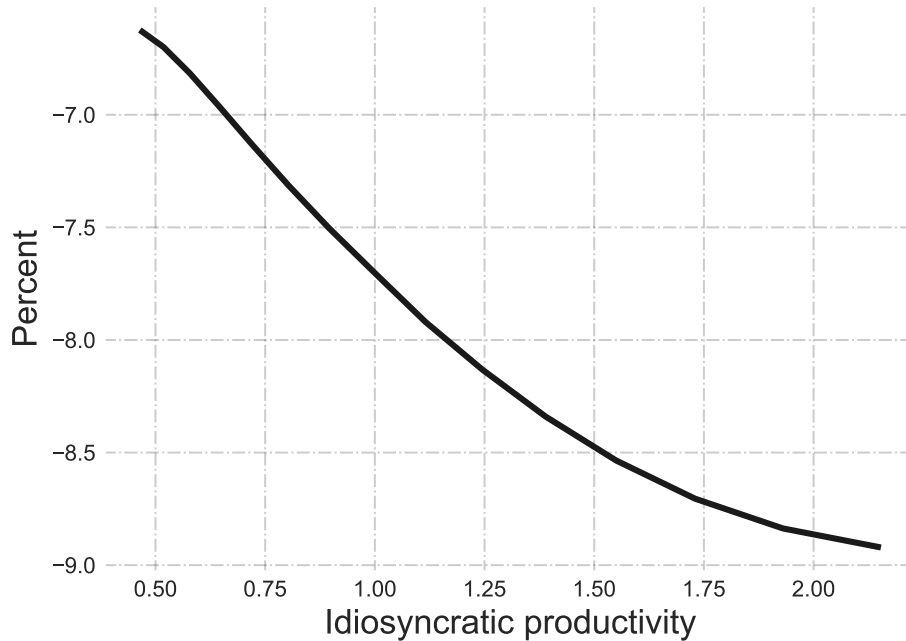


Figure 9: Welfare gain after the tax increase as a function of z

where S_z is the unconditional probability of drawing an idiosyncratic productivity z , and $1_p(a = 0, z)$ is the participation decision of an age 1 individual with 0 assets and productivity z .

Figure 9 plots the welfare gain of moving from the benchmark economy to a 45-percent-tax economy for workers with different idiosyncratic productivity at age one after the workers made their participation decisions. As seen in the figure, the welfare gain decreases with productivity. Notice that the capital-labor ratio goes down after an increase in the tax rates, which leads to a rise in the capital rental rate and a decrease in the wage rate. Consequently, the value of employment goes down. Since higher productivity individuals tend to work, the reduction in the value of employment has more significant effects on them. Therefore, the welfare loss of a tax increase is larger for the high-productivity workers.

The expected welfare gain of a worker before her productivity is revealed, $\tilde{\xi}$, is equal to -7.73% . Overall, this policy results in a welfare loss. On the one hand, the increased rental rate of capital and transfers increases welfare for the capital owners. On the other hand, a reduction in pre- and after-tax wage rates reduces the benefits of employment. Since all individuals start with no wealth, the benefit of an increase in the rental rate of capital is limited. In this quantitative exercise, the welfare loss due to wage reductions dominates.

G Further result of the UI experiment

Figure 10 plots the welfare gain as a function of individual values of z .

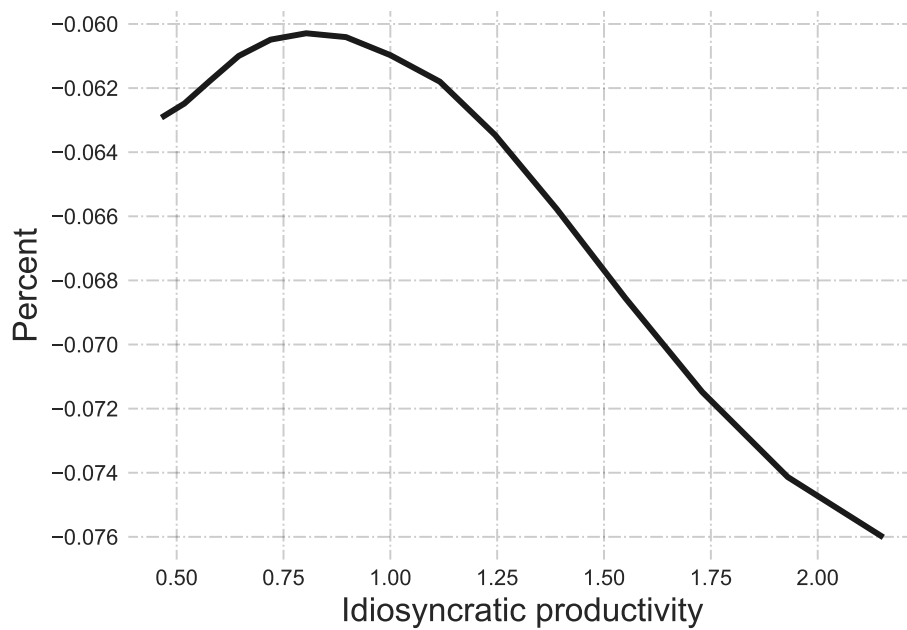


Figure 10: Welfare gain after the increase in the UI replacement rate as a function of z

References

Kuhn, M. and J.-V. Ríos-Rull (2016). 2013 update on the us earnings, income, and wealth distributional facts: A view from macroeconomics. *Federal Reserve Bank of Minneapolis Quarterly Review* 37(1), 2–73.