Growth and Welfare Implications of Sector-Specific Innovations*

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Abstract

I examine the optimal government subsidy of R&D activities when sectors are heterogeneous. To this end, I build an endogenous growth model where R&D drives macroeconomic growth and firm dynamics in two sectors with different characteristics: consumption-goods sector and investment-goods sector. I calibrate the model to U.S. data and study the quantitative properties of the model. By explicitly examining the transition path after the change in subsidy, I highlight the tradeoff between the consumption level in the short run and the long-run growth. I find that the optimal combination of the subsidy rates as a fraction of firm R&D expenditures is 82 percent in consumption sector and 78 percent in investment sector. By moving from the baseline subsidy rates (20 percent in both sectors), the society can achieve 15 percent welfare gain in consumption equivalent terms. The investment sector R&D subsidy generates three quarters of this welfare gain. The annual GDP growth rate increases from 2 percent to 3.3 percent by this change in subsidy. I also analyze the optimal combination of R&D subsidies when the government budget is limited.

Keywords: Endogenous Growth, Innovation, Research and Development, Investment Specific Technological Change

JEL classification: E21, E22, O31,O38, L16

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1 Introduction

Governments support business research and development (R&D) in varying amounts. In 2011, the United States federal government's total support to business R&D was 0.26% of its GDP. The fraction was 0.43% in Korea and .01% in Mexico [OECD (2015)]. Inter-country variation in the government support of R&D suggests that setting the optimal amount of support is not straightforward. In this paper, I characterize the optimal amount of government subsidy to business R&D in a quantitative environment.

I build an endogenous growth model with firm dynamics to analyze the optimal R&D subsidy. I use the model and firm dynamics data to identify inefficiencies in the R&D expenditures of two sectors that have different characteristics: the consumption-goods sector and the investment-goods sector. Next, I characterize the subsidy rates in these sectors that are needed to correct inefficiencies in innovation. I base my model on the seminal work of Klette and Kortum (2004), in which innovation by incumbent and entering firms generates firm dynamics and derives macroeconomic growth. Klette and Kortum show that their model can qualitatively account for various stylized facts about firm dynamics. Hence, I identify the inefficiencies in the firm R&D expenditures with a framework that has a good fit on the firm dynamics data. I extend the Klette and Kortum model by introducing capital stock and by having not only consumption-goods sector but also investment-goods sector. Firm entry and expansion rate differences across these sectors and the sustained decline in the prices of investment goods relative to consumption goods, observed in the data (developed by Gordon (1990), and extended by Cummins and Violante (2002)), imply that in these sectors there are different magnitudes of inefficiency and rates of technological change. A model that takes the heterogeneity of innovative activity across sectors into account will provide more accurate information about the growth and welfare implications of R&D subsidies.

In this paper, an innovation is modeled as an increase in the quality of an existing good in the market along a quality ladder. My two sectors consist of many differentiated products, each of which is produced by a production line. In this setting, firms are simply collections of the production lines they possess. Innovating firms capture the market of the innovated product from the existing producer and earn monopoly rents, which last as long as the innovators hold the blueprints of the highest quality versions of the goods they produce. Firms lose these rents following innovation on the same goods by other firms. Firms, therefore, expand and shrink according to this creative-destruction process, entering the market when they successfully innovate and exiting if they lose the blueprints of all the goods they produce.

In a market economy the creative-destruction process contains inefficiencies, which lead to differences in market economy and social planner innovation rates in each sector. Innovation rate in a sector is defined as the measure of differentiated products innovated at a time over the measure of total products in that sector. An innovator reaps monopoly rents as long as it holds the blueprints of the highest-quality version of the product she innovated. However, the benefit of innovation to the society is the extra production from the innovation, which may be different from the monopoly rents. Also, unlike a limited lifetime of monopoly rents that accrue to the innovator, the social benefit of the innovation lasts forever. On the one hand, an entrepreneur's inability to appropriate all of the consumer surplus it created causes market economy innovation rates to fall below those of social planner levels (the appropriability effect). Also, the limited time of monopoly rents that accrue to the innovator contributes to under investment in innovation (the inter-temporal spillover effect). On the other hand, an entrepreneur does not take into account the profit loss that is imposed on the current producer of the product that they have taken over, and this moves market innovation rates above the socially efficient level (the business-stealing effect). Depending on the sizes of these distortions, market economy innovation rates can be below or above the socially efficient levels. Therefore, a government may be able to employ tax/subsidy systems to correct the distortions in the economy, which increases the welfare of households.

Setting the optimal amount subsidy for each sector requires knowledge of the elasticity of R&D with respect to the subsidy and the magnitudes of externalities in each sector. Various studies have estimated the former using data on firm level R&D expenditures and changes in government subsidy rates [Bloom et al. (2002), Hall et al. (2010), CBO (2005)]. The magnitudes of externalities are not observable. To infer the sizes of externalities in each sector and devise the optimal R&D subsidy system, I identify model parameters related to innovation that are obtained from the model's implications on firm dynamics. Recent literature emphasizes that the firm dynamics data contains important information about the innovation process. Klette and Kortum (2004) show that their model qualitatively generates many empirical facts on firm size distribution and firm growth rates. Lentz and Mortensen (2008), whose model is based on Klette and Kortum, estimate model parameters from Danish data, and the model quantitatively fits related firm dynamics moments.

Differences in the firm dynamics of the two sectors, as observed in the US data, imply that in the two sectors there are different magnitudes of externalities. In the model, the expected lifetime of monopoly rents that accrue to an innovator after successful innovation is linked to the entry rate in the firm's sector (the inter-temporal spillover). Hence, a higher entry rate in the consumption sector suggests a larger inter-temporal spillover effect. The business-stealing effect is about the difference between the profit an innovating firm captures and the net benefit of innovation to the society, including the profit loss the incumbent producer faces after an innovation. The net benefit of innovation to society is the extra production it enables, which is summarized by the size of the quality improvement (the quality ladder step size). The lower the quality ladder step size, the lower the benefit of innovation to society, and the larger the difference between the private and social benefits of an innovation. Therefore, the business-stealing effect is inversely related to the quality ladder step size. The

GDP growth rate and the growth rate of the investment goods price relative to the consumption goods price identifies the quality ladder step size in each sector. The calibration exercise shows that the consumption-goods sector has lower quality ladder step size than the investment-goods sector. Because of the inverse relationship between the quality ladder step size and the business stealing effect, the consumption-goods sector experiences a larger business-stealing effect.

After calibrating my model to the US firm dynamics data, I decompose long-run GDP growth into the contributions of each sector. It turns out that 66 percent of growth is due to innovation activities in the investment-goods sector. A larger quality ladder step size in the investment-goods sector is the main reason for the dominant share of its contribution to growth. This high level of contribution recalls the empirical findings of Sakellaris and Wilson (2004), who report that two thirds of technological progress is attributable to investment-specific technological change. Krusell (1998) develops an endogenous growth model that can account for the decline in the relative price of investment goods. Like me, he attributes approximately half of the consumption growth to investment specific technological change. Moreover, in my analysis, entrants contribute to one-third of consumption growth. My findings regarding the contribution of entrants to growth are comparable to other results reported in the literature. Foster et al. (2001), for example, show that net entry contributes to 25 percent of average TFP growth. Although my methodology differs from the one that by Foster et al. employed, my model leads to growth decompositions that are comparable to theirs.

As explained above, market innovation rates are inefficient. To gauge whether there is under or over investment in innovation, I solve for the social planner problem that is subject to the innovation functions of the firms but can dictate to firms what amount of R&D they should conduct and how much they should produce. Over the long-run, the social planner sets innovation rates that are 9 percentage points higher than the market rates in each sector. Over the long term, the increased innovation rates correspond to a 1.3 percentage point increase in the GDP growth rate. Starting from the balanced growth path of the market economy with 2% GDP growth rate, the social planner immediately increases the GDP growth rate by more than 0.1 percentage point, and then the growth rate gradually increases to its new balanced growth path value of 3.3 percent. However, consumption and GDP follow different trajectories. Like the consumption growth rate, consumption decreases initially as more labor is employed in research. Although the consumption growth rate increases gradually, it remains below its market economy balanced growth path level for some time. Eventually, the consumption growth rate converges with its balanced growth path level of 3.3 percent. Long-run consumption growth outweighs the short-run consumption loss, and the transition from the market economy balanced growth path to the social planner balanced path leads to an almost 15 percent welfare gain to households, as measured in consumption-equivalent terms. Thus, the market is under-investing in innovation, and a benevolent government can increase the welfare of the household by subsidizing R&D.

The amount of resources allocated to R&D at the social planner's balanced growth path is more than three times the market economy resource allocation to innovation. This is similar to what is reported in recent literature that employs related models but different methods. Using Danish data, Lentz and Mortensen (2008) build a model in which firms have persistently different abilities to create higher quality products, and they estimate their model. By using their 2008 paper estimates, Lentz and Mortensen (2015) show that the social planner increases resource allocation to innovation threefold compared to market outcome, which would generate a 21 percent welfare gain, as measured in the tax to social planner consumption. To calculate the welfare gain Lentz and Mortensen compare only the steady states of the market and the social planner economy. The 15 percent welfare gain that I estimate takes into account an additional factor: the transition path. Atkeson and Burstein (2015) develop a method to approximate changes in the path of the economy after a policy-induced change has occurred in the innovation intensity of the economy. Their model nests many models related to innovation, including Klette and Kortum (2004). In a calibration that closely resembles Klette and Kortum model, the social planner increases the amount of resources allocated to innovation three times when a lower discount factor is assumed. In consumption equivalent terms, this would generate a 38 percent welfare gain. When the social planner assumes a higher discount factor, he or she increases the innovation intensity of the economy 11 times, and this would increase the welfare of the society 25 times. To characterize the path of the economy after a policy change, Atkeson and Burstein employ a first-order Taylor approximation around the steady state of the market economy. This restricts their method to the analysis of changes in innovation intensity not as large as the social planner would change.

In my model, in a decentralized environment, the government can increase the welfare of society by employing R&D subsidies and a capital investment subsidy. In my benchmark calibration, subsidizing 82 percent of consumption sector incumbents' R&D expenditures and 78 percent of investment sector incumbents' R&D expenditures generates a welfare gain close to that of the social planner. By providing a capital investment subsidy (an output subsidy would generate the same result), the government fixes the distortions in the capital Euler equation, which is a result of the monopoly power enjoyed by investment goods producers. The government also employs an entry subsidy such that the marginal social cost of entrant innovation equals the marginal social cost of incumbent innovation. This result suggests that government can substantially increase the welfare of a society by heavily subsidizing innovation with constant rates. The government finances these subsidies with lump-sum taxation of households. Similarly, Grossmann et al. (2013) calculate socially optimum time-dependent R&D subsidy rate and find that the welfare loss of setting R&D subsidy rate to its long-run value immediately instead of employing a time-varying R&D subsidy rate is quite low. They also show that the optimal R&D subsidy is approximately 81.5%. Both results are in line with my findings. Akcigit et al. (2016) address optimal R&D policy within a mechanism design framework. They show that when firms are heterogenous in research productivity and there is asymmetric information about research productivity of firms, the optimal subsidy system depends many factors including age of the innovating firms, current and lagged quality of the products of the firms, current and lagged R&D expenditures of the firms. Since there is no asymmetric information in my model and per-good research productivity of firms are constant within a sector, the optimal R&D subsidy is same for all the firms in a sector.

Subsidizing the investment sector produces a larger welfare gain than does subsidizing the consumption sector. Applying a given rate of subsidy to just the investment sector raises welfare by twice as much as applying the same subsidy to just the consumption sector. Two factors contribute to the difference in welfare gains. First, each sector has a similar elasticity of innovation with respect to the user cost of R&D, but the investment sector has a higher innovative step. Hence, any decrease in the user cost would lead to similar changes in innovation rates, but a given change in investment sector innovation leads to a higher consumption growth rate and, hence, a larger welfare gain than would the same amount of change in consumption sector innovation. Second, an increase in the investment sector innovation rate leads to a larger reduction in the price of investment goods. A larger rate of decline in the price of investment goods increases the user cost of capital, which results in a lower accumulation of capital. Consequently, consumption production grows more slowly than it would otherwise. During earlier periods, the investment-sector-subsidized economy has a lower consumption than the consumption sector subsidized economy. In other words, the first factor dominates and the welfare gain of subsidizing investment sector R&D is higher.

To achieve welfare-maximizing innovation rates, the government needs to subsidize innovation at roughly 80 percent. This large subsidy rate is mostly the result of two factors. First, there are significant distortions in the economy. As explained above, using related models, Atkeson and Burstein (2015) and Lentz and Mortensen (2015) find substantial under-investment in innovation in the market economy. Similarly, Jones and Williams (2000) find that the market economy typically under-invests in innovation. Second, R&D subsidies encourage innovation by decreasing the cost of innovation, but they also discourage incumbent innovation by reducing the expected lifetime of an innovation. A higher subsidy leads to a higher firm value, which increases the entry rate. When the entry rate increases, an incumbent firm is more likely to lose its monopoly rents by successfully innovating, and this reduces the expected time period of monopoly rents and the value of innovation (inter-temporal substitution effect increases). Thus, innovation will be discouraged. To compensate for the shortened expected lifetime of innovation, firms needs to be subsidized even more.

I have stressed the importance of firm dynamics and the magnitude of inefficiency differences across the two sectors. It turns out that under the benchmark calibration, the optimal R&D subsidies to sectors are close to one another. This does not mean that the size of each externality across sectors is equal. On the one hand, the inter-temporal spillover effect in the consumption sector is larger than it is in the investment sector. On the other hand, the business-stealing externality is also larger in the consumption sector. Since these two externalities push the innovation rate in opposite directions, the subsidy required to correct these externalities are similar across the two sectors.

This paper is organized as follows. Section (2) describes the model while section (3) calibrates the model. Section (4) decomposes long-run GDP growth into the contributions of the two sectors and the contributions of entering and incumbent firms. Section (5) theoretically characterizes distortions in the economy and numerically compares market outcome to the social planner's equilibrium. Section (6) characterizes the subsidy system that would maximize household welfare. Section (7) concludes.

2 Model

Time is continuous. There are two sectors in the economy: consumption goods and investment goods. Each sector consists of a unit measure of differentiated goods. In turn, each differentiated good has possibly countably many quality levels. Households rent capital to firms, which are owned by the households. Differentiated goods producers engage in research and development (R&D), which results in higher quality levels of existing products in the market.

2.1 Households

An infinitely-lived representative household chooses time paths of consumption, capital holding, investment in capital, and firm holdings to maximize the discounted sum of utility from consumption, $C(t)$,

$$\max \int_0^\infty \exp(-\rho t) \ln C_t dt,$$

subject to the law of motion for capital stock and a budget constraint:

$$\begin{aligned} \dot{K} &= X - \delta K, \\ P_c C + (1 - s_{in}) P_x X + \dot{A} &= RA + wL + rK - T, \end{aligned}$$

where K is the capital stock, X is investment, P_c is the consumption goods price index and normalized to 1, P_x is the investment goods price index, s_{in} is the capital investment subsidy rate, A is total value of the firms, R is the interest rate, w is the wage rate, L is labor supply, r is the rental rate of capital, and T is the lump-sum tax. Henceforth, I will drop time subscripts for notational ease.

Consumption is a CES aggregate of differentiated consumption goods:

$$C = \exp\left(\int_0^1 \ln\left(\sum_{j=0}^{J(\omega)} q^j(\omega)c^j(\omega)\right) d\omega\right), \tag{1}$$

where $q^j(\omega)$ is the quality of version j of product ω, $c^j(\omega)$ is the quantity consumed of version j of product ω, and $J(w)$ is the highest quality version of ω. As seen in Equation (1), households have perfectly substitutable preferences over the different quality adjusted versions of each product. In equilibrium, this formulation leads to the following demand function:

$$c^j(\omega) = \begin{cases} \frac{Z}{p^j(\omega)} & \text{if } \frac{q^j(\omega)}{p^j(\omega)} \geq \frac{q^{j'}(\omega)}{p^{j'}(\omega)} \text{ for all } j' \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where $p^j(\omega)$ is the price of version j of product ω, $Z = P_c C$ is the total consumption expenditure, and $P_c = \exp\left(\int_0^1 \ln\left(\frac{p(\omega)}{q(\omega)}\right) d\omega\right) = 1$.

Investment, X, is also a CES aggregate of differentiated investment goods, which are located in a different interval than consumption goods.

$$X = \exp\left(\int_0^1 \ln\left(\sum_{j=0}^{J(\omega)} q^j(\omega)x^j(\omega)\right) d\omega\right), \tag{3}$$

where $x^j(\omega)$ is the quantity invested in version j of product ω. The corresponding demand function is

$$x^j(\omega) = \begin{cases} \frac{I}{p^j(\omega)} & \text{if } \frac{q^j(\omega)}{p^j(\omega)} \geq \frac{q^{j'}(\omega)}{p^{j'}(\omega)} \text{ for all } j' \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where $I = P_x X$ is the total investment expenditure, and $P_x = \exp\left(\int_0^1 \ln \frac{p(\omega)}{q(\omega)} d\omega\right)$ is the investment price index.

2.2 Firms

A firm is defined by the set of differentiated goods it produces. Each good is produced by a unique production unit. A firm can own countably many production units. It can expand the set of production units by innovating on other goods it currently does not produce. Similarly, it can lose its existing goods to other innovating firms. Furthermore, if a single good producer loses its only production unit, it exits the market. Lastly, entrepreneurs can enter the market by innovating on a good located on the unit interval.

2.3 Innovation by Incumbents

The amount of research labor a firm hires and the number of goods it produces jointly determine its Poisson innovation arrival rate, β. Innovation is not directed. The good that is innovated is randomly drawn from a uniform distribution on the unit interval of goods in the market. A firm innovates only in the sector that it currently operates in. An innovation increases the quality of the good by an exogenous factor of $\lambda > 1$. This factor is the quality ladder step size and represents the innovativeness of a firm. Throughout the paper, the term *innovativeness* will be used to signify the factor by which the quality of a product increases after a successful innovation. More innovative firms can increase the quality of a good by a larger factor. The level of innovativeness varies by sector but is invariant across firms within each sector. After a successful innovation, the innovator and the firm which has the blueprints of producing the second highest quality version (runner-up) of the good engage in Bertrand competition. The innovator charges a price equal to λ times marginal cost of production of second highest quality version of the good. Since consumers have infinitely elastic preferences over the quality adjusted varieties of a good, the innovator takes over the market. The innovator expands by one good, the runner-up shrinks by one good.

A firm currently producing m goods and hiring l_R units of labor for the purpose of research innovates at a rate $\varphi(m, l_R) = \beta$, where $\varphi(\cdot)$ is a constant returns to scale production function, increasing in both arguments, and strictly concave in l_R. Firms with experience in innovation, particularly those that retain their products despite innovation by other firms, are better at producing ideas. The number of goods in the production function is a proxy for a firm's experience in innovation.

2.4 Consumption Goods Producers

I define the problem of a differentiated consumption good producer in two steps. First, I define the static problem: how much to produce, and demand for factor inputs. After solving this problem and establishing the profit from production, I turn to the dynamic problem: how much to invest in R&D to maximize the value of the firm.

Each production unit of a firm has a Cobb-Douglas production function with capital elasticity α. Production of each unit is independent of other production units a firm may possess. Hence, each production unit solves the following cost minimization problem:

$$\min_{l_c, k_c} wl_c + rk_c \quad \text{subject to} \quad k_c^{\alpha} l_c^{1-\alpha} = c,$$

where w and r are the market wage and capital rental rates. The resulting cost function, $C((w, r), c) = \frac{r^{\alpha} w^{1-\alpha} c}{\tilde{\alpha}}$, with $\tilde{\alpha} \equiv \alpha^{\alpha}(1-\alpha)^{1-\alpha}$, is common across all the production units in a sector. As a result, Bertrand competition yields a price $p^j = \lambda_c \frac{r^{\alpha} w^{1-\alpha}}{\tilde{\alpha}}$ for differentiated product j. Using this price and the demand function for differentiated goods yields the profit of a differentiated good producer:

$$\pi = pc - C((w, r, c)) = \left(1 - \frac{1}{\lambda_c}\right) Z. \tag{5}$$

Note that profits do not vary across differentiated goods in a sector.

Turning to the dynamic problem, I use the profit function in (5) to derive the firm's value function, which can be expressed either as a function of the level of research labor or, more conveniently, the level of innovation arrival rate per good. Let $\phi(\beta, m)$ denote the level of l_R implicitly defined by $\varphi(m, l_R) = \beta$. Since $\varphi(\cdot)$ is strictly increasing in l_R, and homogenous of degree one, $\phi(\cdot)$ is well-defined and homogenous of degree one and convex in β. Set $\phi_c(b_c, m) = m\chi_c b_c^\gamma$, where $b_c = \beta/m$, and $\chi_c > 0$ is a scale parameter.

The Bellman equation of the firm on the balanced growth path is

$$\begin{aligned} RV(m, Z) &= \max_{b_c \geq 0} \left\{ \left(1 - \frac{1}{\lambda_c}\right) mZ - (1 - s_c^i) w \phi_c(b_c, m) + \frac{\partial V(m, Z)}{\partial Z} \dot{Z} \right. \\ &+ \left. m b_c [V(m+1, Z) - V(m, Z)] + m\tau_c [V(m-1, Z) - V(m, Z)] \right\}, \end{aligned}$$

where s_c^i is the rate of R&D subsidy for the consumption sector incumbents, and τ_c is the equilibrium Poisson innovation arrival rate in the consumption sector. Given that firm profits, and R&D expenditures are linear in the number of goods, I conjecture that $V(m, Z) = \nu_c m Z$ for some $\nu_c > 0$ and verify this claim. Inserting the guess yields

$$R\nu_c m Z = m \left(1 - \frac{1}{\lambda_c}\right) Z - (1 - s_c^i) w m \chi_c b_c^\gamma + \nu_c m Z g_Z + m b_c \nu_c Z - m\tau_c \nu_c Z,$$

where b_c is the optimal innovation intensity, and $g_Z \equiv \frac{\dot{Z}}{Z}$ is the growth rate of household consumption expenditure.

Rearranging and simplifying the above equation leads to the following solution to the Bellman eqution:

$$(R - g_Z + \tau_c - b_c)\nu_c = \left(1 - \frac{1}{\lambda_c}\right) - (1 - s_c^i)\chi_c b_c^\gamma \frac{w}{Z}. \tag{6}$$

The first order condition for innovation arrival rate per product is:

$$(1 - s_c^i) w \gamma \chi_c b_c^{\gamma - 1} = \nu_c Z. \tag{7}$$

Equation (7) establishes that innovation arrival rate per product is independent of firm size or the number of goods a firm produces. In my solution of the stationary equilibrium, the growth rate of consumption expenditures, g_Z, is constant and equal to the growth rate of wages, g_w. Appendix B solves the problem and shows the equality of g_Z and g_w. All this, together with Equation (6), verify that the value function is indeed linear in m.

Entrepreneurs can enter the market by innovating on a product. Like incumbents, they hire research labor to develop better qualities of products. An entrepreneur must hire $\xi_c(z_c, \bar{z}_c) \equiv \psi_c \chi_c z_c \bar{z}_c^{\gamma-1}$ units of labor to secure a z_c Poisson innovation rate, where, $\psi_c > 0$ is a parameter to differentiate the cost of incumbent and entrant innovation, and $\bar{z}_c$ is the entry rate in the market. As more entrants try to enter to the market, it requires more effort to develop a successful product. This formulation is a reduced form of the limited availability of venture capital to entrepreneurs. The value of entry is therefore

$$RV_E = \max_{z_c \geq 0} \{-(1 - s_c^e) w \xi_c(z_c, \bar{z}_c) + z_c[V(1, Z) - V_E]\},$$

where s_c^e is entry subsidy rate (i.e. entrant innovation subsidy) in the consumption sector. Free entry drives down the value of entry to zero. Hence, in equilibrium

$$(1 - s_c^e) w \psi_c \chi_n \bar{z}_c^{\gamma-1} = V(1, Z). \tag{8}$$

2.5 Investment Goods Producers

Firms in this sector share the same Cobb-Douglas production function with capital elasticity α. The profit of a production unit, $\pi = \left(1 - \frac{1}{\lambda_x}\right) I$, is derived in a manner analogous to that of the consumption goods producers. The value of a firm currently producing m goods, $V(m, I)$, solves the Bellman equation

$$\begin{aligned} RV(m, I) \quad = \quad & \max_{b_x \geq 0} \left\{ \left(1 - \frac{1}{\lambda_x}\right) mI - (1 - s_x^i) w \phi_x(b_x, m) + \frac{\partial V(m, I)}{\partial I} \dot{I} \right. \\ & + \left. m b_x [V(m+1, I) - V(m, I)] + m \tau_x [V(m-1, I) - V(m, I)] \right\}, \end{aligned}$$

where s_x^i is the R&D subsidy rate for the investment sector incumbent firms. Guessing and verifying that the value function of investment goods producers is also linear in m, the following equations characterize the value function and first order condition for the optimal innovation arrival rate per product:

$$(R - g_I + \tau_x - b_x)\nu_x = \left(1 - \frac{1}{\lambda_x}\right) - (1 - s_x^i)\chi_x b_x^\gamma \frac{w}{I}, \tag{9}$$

$$(1 - s_x^i) w \gamma \chi_x b_x^{\gamma-1} = \nu_x I. \tag{10}$$

Entrants, on the other hand, have the following problem:

$$RV_E = \max_{z_x \geq 0} \{-(1 - s_x^e) w \xi_x(z_x, \bar{z}_x) + z_x[V(1, I) - V_E]\},$$

where s_x^e is the entry subsidy in investment sector. Free entry implies

$$- w \psi_x \chi_x \bar{z}_x^{\gamma-1} = V(1, Z). \tag{11}$$

2.6 Equilibrium

A symmetric balanced growth path competitive equilibrium is defined by a tuple of firm decisions $\{k_{i,t}, l_{i,t}, l_{R,i,t}, b_{i,t}, z_{i,t}, \tau_{i,t}, c_t, x_t\}$, where $i = c, x$ represents consumption and investment sectors, a tuple of household decisions $\{c_t, x_t, C_t, X_t, K_t\}$, a tuple of prices $\{w_t, r_t, R_t, p_{c,t}, p_{x,t}, P_{c,t}, P_{x,t}\}$, aggregate expenditures $\{Z_t, I_t\}$, average quality levels in each sector, $\{Q_{c,t}, Q_{x,t}\}$, and value of production units per aggregate expenditure in a firm's sector, $\{\nu_c, \nu_x\}$. In equilibrium the following conditions hold.

- $\{p_c, p_x\}$ are the Bertrand equilibrium prices of highest quality products.
- Given prices of differentiated goods and household demand functions (2) and (4) k_c, l_c and k_x, l_x solve the firm cost-minimization problems in the consumption and investment sectors.
- Given prices and nominal aggregate expenditures, $\{\nu_c, b_c, z_c\}$ solve equations (6), (7), (8) for $i = c$, and $\{\nu_x, b_x, z_x\}$ solve equations (9), (10), (11) for $i = x$.
- Innovation rate in a sector is equal to sum of incumbent and entrant innovation rates: $\tau_{i,t} = z_{i,t} + b_{i,t}$, where $i = c, x$.
- Given prices, $\{c_t, x_t, C_t, X_t, K_t\}$ are the balanced growth path values of the household optimization problem.
- The labor market clears: $l_c + l_x, +\chi_c b_c^\gamma + \chi_x b_x^\gamma + \psi_c \chi_c z_c^\gamma + \psi_x \chi_x z_x^\gamma = L$,
- The capital market clears: $K_t = k_{c,t} + k_{x,t}$,
- Nominal expenditures, $\{Z, I\}$, grow at the same rate.
- Average quality levels of industries are $Q_c = \exp\left(\int_0^1 \ln\left(q(\omega)\right) d\omega\right)$, $Q_x = \exp\left(\int_0^1 \ln\left(q(\omega)\right) d\omega\right)$.

This equilibrium is discussed in detail in Appendix B.

3 Calibration

I classify industries into consumption goods producers and investment goods producers following the methodology applied by Huffman and Wynne (1999). Specifically, I classify an industry as a consumption goods producer if, according to the economy's input-output table, household consumption of the industry output is larger than sum of the industry output added to inventory and output used by other firms. Similarly, investment goods industries are those whose output is used mostly by other firms. Appendix A specifies the industries in each group.

Table 1 reports the parameters that are calibrated independently from the data or taken from other papers. A unit length of time in the model is considered as a year in the data. The elasticity of output with respect to capital is chosen as 0.33. The depreciation rate, δ, is calibrated to have a 5% annual deprecation rate [KLEMS data on U.S.], and the discount rate is targeted to have a 0.97 annual discount factor.

Table 1: Externally calibrated parameters

	Parameter	Value
Elasticity of output w.r.t capital	α	.33
Depreciation Rate	δ	.06
Discount Rate	ρ	.03
Curvature of R&D cost function	γ	2.5
R&D subsidy, consumption incumbents	s_c^i	.2
R&D subsidy, investment incumbents	s_x^i	.2

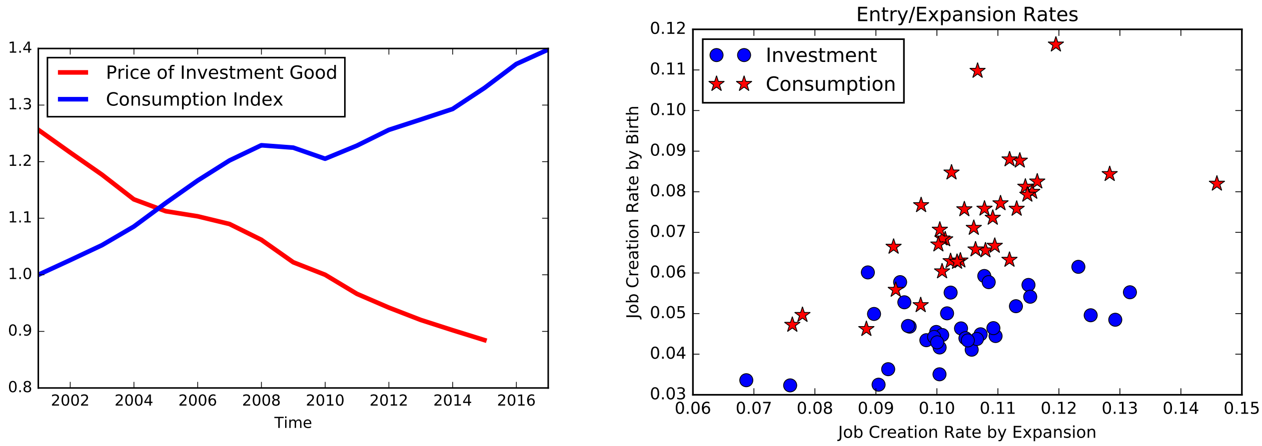
To calibrate the curvature of the R&D cost function, γ, I target price elasticity of R&D with respect to its user cost estimated by Bloom et al. (2002). They estimate both short-run and long-run elasticity of R&D with respect to its user cost. Short–run elasticity, the immediate effect of user cost changes, is estimated as 0.35. This value corresponds to a γ value of 3.85 in my model. However, firms' R&D expenditures are highly persistent. A change in user cost at the current period affects R&D in all subsequent periods. Bloom at al. estimate a long-run elasticity, sum of R&D changes in all subsequent periods, as approximately 1. This elasticity corresponds to a γ value of 2 in my model. However, neither of these estimates correspond exactly to my model. In the model, firms make R&D decision for each period and get the benefit of R&D immediately. In reality, firms commit to R&D for certain periods of time, but not indefinitely. Therefore, I choose $\gamma = 2.5$, a number that corresponds approximately to midway between the short-run and long-run elasticities. R&D subsidy rates for incumbent firms in the two sectors are chosen as 0.2 following the R&D tax credit rate in the US [Bloom et al. (2002)].

Table 2: Targets

	Variable	Data	Model
Entrant innovation rate, consumption	z_c	.06	.06
Entrant innovation rate, investment	z_x	.04	.04
Incumbent innovation rate, consumption	b_c	.1	.1
Incumbent innovation rate, investment	b_x	.09	.09
Consumption growth rate	g_C	.02	.02
Growth rate of investment good prices relative to consumption good prices	g_{P_x}	-.02	-.02

Other parameters of the model are calibrated by using the implications of the model on firm entry and expansion rates. The innovation rate by entrants corresponds to the firm entry rate — the measure of entering production units over the total measure of production units in the sector. The innovation rate by incumbents corresponds to firm expansion rates — the measure of production units captured by incumbent firms over the total measure of production units in the sector. Further, since each production unit in a sector employs the same amount of labor, the innovation rate by entrants is equal to the number of jobs created by entering firms over the total employment in that industry (job creation rate by birth). Similarly, the innovation rate by incumbents is equal to the number of jobs created by expanding firms over the total employment in that industry, which is called job creation rate by expansion. Statistics of United States Businesses (SUSB) provides job creation rates by birth and job creation by expansion of establishments for 4 digit NAICS. I use SUSB data from 1999 to 2012 to compute average job creation rates by birth and expansion for the industry classification of this paper. The first row of Table 2 shows these rates which I target as innovation by entrants and incumbents in each sector.

Figure 1: Industry Dynamics



The model also has implications on the growth rates of consumption and the relative price of investment goods, equations (12) and (13). Technological progress in each industry contributes to the growth rate of consumption, whereas the growth rate of the relative price of investment goods depends on the difference of technological progress in each sector.

$$g_C = \tau_c \ln \lambda_c + \frac{\alpha}{1-\alpha} \tau_x \ln \lambda_x \tag{12}$$

$$g_{P_x} = \tau_c \ln \lambda_c - \tau_x \ln \lambda_x \tag{13}$$

The consumption growth rate is targeted as 2%, the historical average of the US consumption growth rate, depicted in the left panel of Figure 1. The growth rate of the price of quality adjusted investment goods is approximately -2%. [Gordon (1990), Cummins and Violante (2002) and DiCecio (2009)] Therefore, the innovativeness of sectors (λ_c and λ_x) is identified using Equations (12) and (13) and target rates for the consumption growth rate, the change in the relative price of investment goods, and the innovation rates in each industry.

The other parameters of the model seen in Table 3 are calibrated to make the model moments match with the target moments in the data. The relative costs of entry ψ_x, ψ_c, in each sector are identified by job creation by birth over job creation by expansion rate in these industries. The right panel of Figure 1 shows the job creation rates by birth and expansion for consumption and investment goods sector. Each star or circle is an annual observation from the data. Stars represent the consumption

Table 3: Internally calibrated parameters

	Parameter	Value
Quality ladder step size, investment	λ_x	1.23
Quality ladder step size, consumption	λ_c	1.04
R&D cost function parameter, investment	χ_x	10.93
R&D cost function parameter, consumption	χ_c	5.73
Entry cost function parameter, investment	ψ_x	6.75
Entry cost function parameter, consumption	ψ_c	4.30

goods industry observations and circles the investment goods industry observations. As seen in the graph, job creation rate by birth over job creation by expansion is higher in consumption industries. This data results in a lower ψ value of the consumption sector. Overall, the results of the calibration exercise indicate that: 1) the quality ladder step size of investment goods is higher than that of consumption goods ($\lambda_x > \lambda_c$), 2) innovation in the investment goods sector is more costly ($\chi_x > \chi_c$). 3) innovation is costlier for entrants ($\psi_x, \psi_c > 1$), and 4) entry is more costly in the investment sector ($\psi_x > \psi_c$).

4 Growth Decomposition

Consumption growth in the long-run is a result of innovative activities in the two sectors. As described in Equation (12), consumption growth rate can be decomposed into contributions of technological progress in consumption goods and technological progress in investment goods. Using the definition of the total innovation rate, which is the sum of entrant innovation and incumbent innovation, consumption growth rate can be further decomposed into contributions of entrants and incumbents:

$$\begin{aligned} g_C &= (z_c + b_c)\ln\lambda_c + \frac{\alpha}{1-\alpha}(z_x + b_x)\ln\lambda_x \\ g_C &= \underbrace{z_c \ln\lambda_c}_{\substack{\text{Consumption}\\ \text{Entrants}}} + \underbrace{b_c \ln\lambda_c}_{\substack{\text{Consumption}\\ \text{Incumbents}}} + \underbrace{\frac{\alpha}{1-\alpha} z_x \ln\lambda_x}_{\substack{\text{Investment}\\ \text{Entrants}}} + \underbrace{\frac{\alpha}{1-\alpha} b_x \ln\lambda_x}_{\substack{\text{Investment}\\ \text{Incumbents}}}. \end{aligned} \tag{14}$$

Equation (14) decomposes the growth rate by sectors and entrants/incumbents. Table 4 shows each element's contribution as a percentage of the consumption growth rate. The investment sector contributes 66% percent of growth, whereas the consumption sector contributes 34%. The contribution of the investment sector in my estimates is comparable to estimates of Sakellaris and Wilson (2004), who empirically find that embodied technological change in investment goods contributes two thirds of macroeconomic growth.

Entrants contribute approximately one third to growth. Remember, in the model, the innovativeness of entrants and incumbents are the same. Hence, the difference of entrants and incumbents in terms of contribution to growth stems mainly from different entry and expansion rates. Similar to my results, by using the Census of

Table 4: Consumption Growth Decomposition

	Consumption	Investment	Total
Entrant	13%	20%	33%
Incumbent	21%	46%	67%
Total	34%	66%	

Manufacturers data from 1977 to 1987, Foster et al. (2001) show that net entry contributes one quarter of multi–factor productivity growth. Overall, investment sector incumbents contribute the most to growth and consumption sector entrants contribute the least. Intuitively, most of the growth comes from companies producing better machines, and less comes from consumption sector entrants like new restaurants.

5 Optimality of Innovation

As a characteristic of Schumpeterian creative–destruction type models, the competitive equilibrium innovation rate may not be socially optimal. An innovating firm improves the quality of an existing good, destroys the profit accrued by the incumbent producer and gains monopoly power on production of the product that it innovated. While deciding the amount of R&D to conduct, it considers the monopoly profits that it will accrue until another firm innovates on that good and captures the product. However, the social benefit of an innovation goes on forever since every innovator improves the quality upon the existing quality level. Also, innovators ignore the profit loss of the existing producer of the good. Therefore, the competitive equilibrium innovation rate is generically inefficient. After defining the social planner problem, I will discuss each externality further in Section 5.2.

In order to identify how the externalities affect the economy, I define and solve the social planner's problem. Then, I compare the competitive equilibrium first order conditions with the social planner first order conditions, and discuss the differences caused by externalities.

5.1 Social Optimum

The social planner problem (SP) can be divided into two parts: 1) a static problem where a given level of total innovation in a sector is allocated to entrants and incumbents, and 2) a dynamic problem where the time paths of labor, capital and innovation are determined.

In the static problem, the social planner minimizes the research cost of a fixed aggregate innovation in a sector by choosing incumbent entry and innovation rates:

$$\min_{z_i, b_i} \psi_i \chi_i z_i^\gamma + \chi_i b_i^\gamma \quad \text{subject to} \quad z_i + b_i = \tau_i,$$

where z_i is the entry rate, b_i is the innovation rate by incumbents, τ_i is the aggregate innovation rate, and $i = c, x$ represents sectors. Note that the social planner takes into account the externality created by entrants on each other. The resulting cost function (in labor units) for a sector is

$$C_i(\tau_i) = \frac{\psi_i \chi_i \tau_i^{\gamma}}{\left(1 + \psi_i^{1/(\gamma-1)}\right)^{\gamma-1}}. \tag{15}$$

The economy-wide research cost function is the sum of innovation costs across sectors,

$$C(\tau_c, \tau_x) = \frac{\psi_c \chi_c \tau_c^{\gamma}}{\left(1 + \psi_c^{1/(\gamma-1)}\right)^{\gamma-1}} + \frac{\psi_x \chi_x \tau_x^{\gamma}}{\left(1 + \psi_x^{1/(\gamma-1)}\right)^{\gamma-1}}.$$

Using the research labor cost function found in the static problem, the social planner then maximizes the discounted sum of utility from consumption:

$$\max \int_0^{\infty} e^{-\rho t} \ln(K_{c,t}^{\alpha} L_{c,t}^{1-\alpha} Q_{c,t}) dt \quad \text{subject to}$$

the resource constraints of capital, $K_{c,t}+K_{x,t} = K_t$, and labor, $L_{c,t}+L_{x,t}+C(\tau_{c,t}, \tau_{x,t}) \leq 1$, the law of motion for capital stock, $\dot{K}_t = K_{x,t}^{\alpha} L_{x,t}^{1-\alpha} Q_{x,t} - \delta K_t$, the average quality levels in each sector, $Q_{c,t} = \exp\left(\int_0^1 \ln\left(q_t(\omega)\right) d\omega\right)$, $Q_{x,t} = \exp\left(\int_0^1 \ln\left(q_t(\omega)\right) d\omega\right)$, and laws of motion for the technology index of the consumption sector, $\frac{\dot{Q}_{c,t}}{Q_{c,t}} = \tau_{c,t} \log \lambda_c$, and the investment sector, $\frac{\dot{Q}_{x,t}}{Q_{x,t}} = \tau_{x,t} \log \lambda_x$.

5.2 Distortions

Innovative activity leads to various distortions in the market equilibrium conditions relative to the social optimum. First, an improvement in the quality level of a good gives market power to the innovator, i.e. she can charge a markup over the marginal cost of production. Second, quality improvements occur over existing innovations ('standing on the shoulders of giants'). Hence, an innovation increases the quality level of a good forever, but the innovator gets the benefit for a limited time, until the next innovation on the good. Third, innovation destroys the profit accruing to the incumbent ('business stealing'). Fourth, the cost of entry into the market by an entrepreneur increases with the measure of total innovative activity by entrants. The first order condition for innovation rates (denoted with by ' ^ ') in competitive equilibrium, and first order condition for social planner innovation (denoted with ' * ') are as follows:

$$c'(\hat{b})w = \frac{1(\pi - c(\hat{b})w)}{\rho + \tau - b}, \tag{16}$$

$$c'(b^{*})F_L(K, L, Q) = \frac{\ln(\lambda)F(K, L, Q)}{\rho}, \tag{17}$$

where $c(\cdot)$ is the R&D cost function defined in Equation 15, and $F(\cdot)$ sector-level production function. For the sake of simplicity of notation, I dropped sector subscripts. Equations (16) and (17) hold for each sector. Following Aghion and Howitt (1992), I compare Equations (16) (competitive equilibrium first order condition) and (17) (social planner FOC) to understand the effect of these distortions on innovation level. These FOCs equate the marginal cost of innovation to the discounted benefit of innovation. The marginal cost of the innovation in the competitive equilibrium is $c'(b)w$, while the marginal cost in the SP problem is $c'(b)F_L(K, L, Q)$. Since firms have monopoly power, the marginal product of labor may differ from the wage rate. This *monopoly-distortion effect* causes the competitive equilibrium innovation level to exceed the SP level (Aghion and Howitt (1992)).

Second, the private flow benefit of innovation is the monopoly profit minus research cost, $\pi - c(b)w$, whereas the social benefit is total output $F(K, L, Q)$, so that b^* exceeds $\hat{b}$ [*Approbriability*]. Third, as a result of innovation, the monopolist takes over the market for the good, and it does not consider the loss the incumbent incurs. Hence, we have '1' in front of π. However, the social planner considers the change in utility as a result of collective innovation. Hence it has $\log \lambda$ in front of $F(\cdot)$. This *business-stealing effect* leads to a higher level of private innovation. Fourth, the private innovator accrues the benefits as long as she has the monopoly power over the good. Therefore, she discounts the profits at a rate $\rho + \tau - b$. However, the benefits of innovation accrue to society forever, since the quality increase lasts eternally. This *inter-temporal spillover effect* yields higher social planner innovation levels.

The Euler equations in the market economy and social planner are

$$\frac{1}{\lambda_x} \alpha \hat{K}_x^{\alpha-1} \hat{L}_x^{1-\alpha} Q_x - \delta - \rho = \frac{1}{1-\alpha} \hat{\tau}_x \ln \lambda_x, \tag{18}$$

$$\alpha {K^*}_x^{\alpha-1} {L^*}_x^{1-\alpha} Q_x - \delta - \rho = \frac{1}{1-\alpha} \tau_x^* \ln \lambda_x. \tag{19}$$

Monopoly pricing distorts the price of the investment good, leading to differences in the Euler Equations (18) and (19): $1/\lambda_x$ appears in front of the marginal product of capital in the investment goods sector in competitive equilibrium, but not in the social planner equation. This distortion leads to less private capital than the social optimum. However, innovation in investment goods also affects the change in relative price of investment goods. The higher the innovation, the greater the decline in price of investment goods. The greater pace of decline in the price of investment goods makes acquiring capital in initial periods costlier. Hence, regimes that have a higher innovation in investment goods have lower level of capital.

Though it is not the focus of this paper, there is another externality created by the entry process, namely entrants do not internalize the extra entry cost they impose on other entrants. Equation (20) shows the competitive equilibrium first order condition for allocation of innovation between entrants and incumbents whereas Equation (21) shows the social planner allocation of innovation. In the social planner allocation marginal costs of entry and incumbent innovations are equated, but not in the competitive equilibrium. This leads to a more than optimal entry rate.

$$\psi\chi\hat{z}^{\gamma-1} = \gamma\psi\chi\hat{b}^{\gamma-1} \tag{20}$$

$$\gamma\psi\chi z^{*\gamma-1} = \gamma\psi\chi b^{*\gamma-1} \tag{21}$$

Lastly, since capital and labor markets are competitive, the only distortion in the factor demand equations comes from monopoly pricing of the goods. Equating relative factor prices across sectors, I get the undistorted capital labor ratios. Equation (22) is identical in market equilibrium and in the social planner allocation:

$$\frac{1-\alpha}{\alpha}\frac{K_x}{L_x} = \frac{1-\alpha}{\alpha}\frac{K_c}{L_c}. \tag{22}$$

5.3 Under Investment in Innovation

Of the distortions identified above, appropriability and inter–temporal spillover effects cause the economy to under-invest in innovation whereas business stealing and monopoly distortion cause the economy to over-invest in innovation. Whether the economy under-or over-invest in innovation depends on the parameters of the model. In this economy, it is under-investment as shown in Table 5. Column 1 shows the competitive equilibrium consumption growth rate, innovation rates in sectors and discounted capital stock, $\tilde{K} = \frac{K}{Q_x^{1/1-\alpha}}$, which is the capital stock level at the steady state of the economy where variables are discounted accordingly with the technology indices. Column 2 shows the social planner values. Socially optimal innovation rates in both sectors are 9 percentage points higher than competitive equilibrium rates. These higher innovation rates make the economy grow 1.3 percentage point faster under the social planner. In a similar exercise, using a Schumpeterian creative destruction model whose parameters are estimated using Danish firm level data, Lentz and Mortensen (2015) find that the optimal growth rate is twice as much as the competitive equilibrium growth rate.[1]

Later, I discuss the welfare implications of R&D subsidies that push the economy towards social planner allocations. The change in consumption growth rate will play an important role in generating welfare gain. The other important factor that needs to be analyzed is capital stock. Removing the monopoly distortion in the capital Euler equation (18) leads to higher capital accumulation, whereas higher user cost of capital, resulting from higher innovation in investment goods, would result in lower accumulation of capital. Here, the latter force dominates and steady state capital stock of the social planner is lower than capital stock in the market economy. A transition of the economy from competitive equilibrium innovation levels to social planner innovation levels would cause labor allocated to consumption good production to decrease. On the other hand, if the amount of capital invested also decreases then

[1] They consider a model where firms in a sector have different innovativeness which evolve according to a Markov Process.

some of the labor allocated to investment good production can be used in consumption good production or research. This will create extra welfare gain in the economy.

Table 5: Competitive Equilibrium vs Social Planner

	CE	SP		
			CE τ_c	CE τ_x
g_C	0.020	0.033	0.030	0.025
τ_c	0.160	0.252	0.160	0.277
τ_x	0.130	0.219	0.230	0.130
$\tilde{K}$	3.027	2.390	2.477	3.671

Notes: Column 1 (CE) shows the competitive equilibrium values, column 2 (SP) the social planner values, column 3 (CE τ_c) the social planner values when she is restricted to have competitive equilibrium innovation rate in consumption sector, and column 4 (CE τ_x) when the social planner is restricted to have competitive equilibrium innovation rate in investment sector.

After substituting the functional forms and rearranging Equations (16) and (17), we get Equations (23) and (24) below. Now, inter-temporal spillover effect is the difference between the denominators of the right-hand sides of the equations, and the combination of appropriability, business-stealing, and monopoly distortion effects is represented by the difference between the numerators of the right-hand sides of equations.

$$\frac{1}{1-\gamma}\chi_j b_j^{\gamma/(1-\gamma)} = \frac{\frac{(\lambda_j-1)L_j}{(1-\alpha)(1-s_{ij})} - \chi_j b_j^{1/(1-\gamma)}}{\rho + z_j} \tag{23}$$

$$\frac{1}{1-\gamma}\chi_j b_j^{\gamma/(1-\gamma)} = \frac{\frac{\ln\lambda_j L_j}{1-\alpha}}{\rho}, \quad j = c, x. \tag{24}$$

To understand the relative importance of the distortions in the two sectors, I conduct the following exercise which is reported in Table 6. When only inter-temporal spillover is corrected, and holding the other values fixed, the incumbent innovation rate in the consumption-goods sector doubles and becomes more than the socially efficient innovation rate. The incumbent innovation rate in the investment-goods sector also increases substantially, but it does not go above the social planner level. When business-stealing is corrected, the incumbent innovation rate in the consumption-goods sector declines 90 percent to 0.01. Whereas, incumbent innovation rate in investment-goods sector reduces by 66 percent to 0.03. This exercise shows that both inter-temporal spillover and business-stealing effects are stronger in the consumption-goods sector.

To gauge the relative importance of innovations in the two sectors, I conduct the following exercises which are shown in columns 3 (CE τ_c) and 4 (CE τ_x) of Table 5. In the exercise depicted in column 3, I solve the social planner problem while constraining the innovation rate in the consumption sector to the competitive equilibrium value. Now, the social planner allocates more labor to innovative activity in the investment

Table 6: Incumbent Innovation Rates

	Consumption	Investment
Competitive equilibrium	.10	.09
Correct inter-temporal spillover	.21	.16
Correct business-stealing	.01	.03
Social Planner	.18	.17

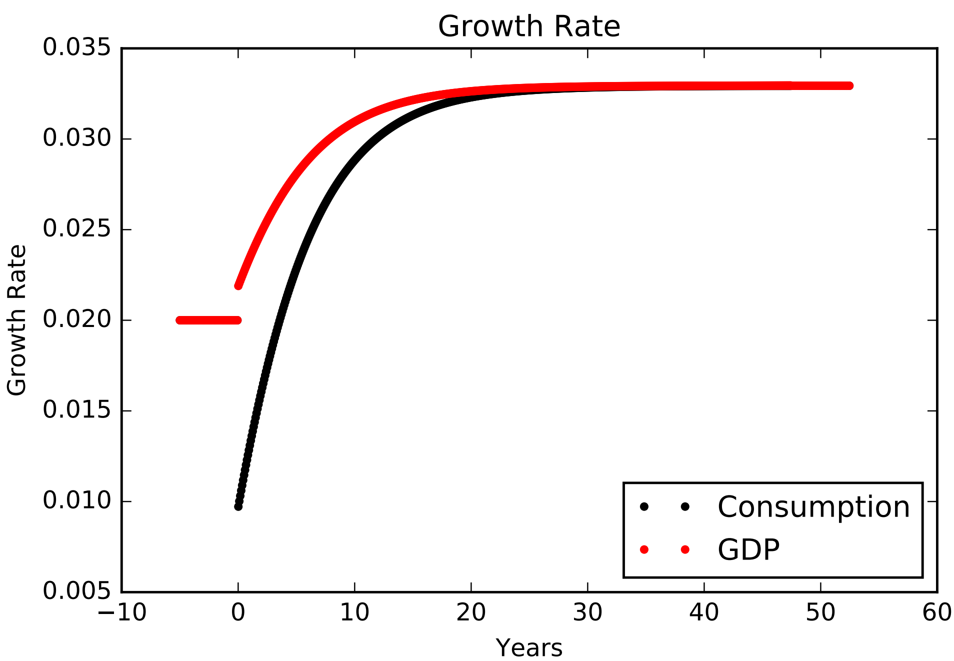
Notes: Competitive equilibrium incumbent firm innovation rates and when certain externalities are corrected.

sector and reaches a consumption growth rate close to the unconstrained problem rate. However, in a similar exercise where the investment sector innovation rate is constrained to its competitive equilibrium level instead of the consumption sector, the consumption growth rate is 0.8 percentage points lower than the social planner rate (reported in column 4). Though the social planner increases the innovation rate in the consumption sector, it is not enough to compensate purge in the investment sector innovation. These exercises point out that the socially optimum growth rate is significantly higher than the growth rate in the market economy. Also, it is the under-innovation in the investment sector that leads to a large gap between competitive equilibrium and the socially optimum growth rates.

The change in steady state capital stock level is also in line with the analysis comparing the competitive equilibrium with the social planner. When restricting the investment sector innovation to CE levels, steady state capital stock increases. Holding the innovation rate fixed in this sector maintains the user cost of capital at the market economy level. However, the social planner still corrects the monopoly pricing of the investment good. As a result, steady state capital stock increases relative to the competitive equilibrium. However, when the consumption sector innovation is restricted, and the social planner is free to the choose investment sector innovation, capital stock is less than the competitive equilibrium level but higher than the social planner level. The former is expected. The second one may seem counter to my arguments above. The investment sector innovation rate in the constrained social planner's solution is higher than the unconstrained social planner, and hence the user cost of capital is higher. We would expect a lower accumulation of capital, but we observe a higher accumulation of capital. The reason is: a reduction in the consumption sector innovation frees up some labor which can be allocated to investment good production. This increased labor in the sector leads to higher marginal product of capital. That is why we see an increase in capital accumulation relative to the unconstrained social planner problem. I will return to the importance of capital accumulation when I discuss the welfare implications of innovation subsidies.

Analyzing just the balanced growth path does not give the whole picture of welfare and growth implications of the socially optimal innovation. Figure 2 depicts GDP and the consumption growth rate of the social planner equilibrium starting from the balanced growth path of the market economy. Section 6 explains the solution method in detail. Since this is a two sector economy, GDP of the social planner is calculated as if the relative price of the two sectors follows the market economy pricing. The social planner allocates more labor to research and consumption decreases immediately. As the technological progress rate increases, so does the consumption growth rate. However, it takes years for the economy to have a higher consumption growth rate than the market economy balanced growth path. After three years, the consumption growth rate surpasses 2 percent and eventually reaches the long-run rate of 3.3 percent. The GDP growth rate, on the other hand, increases initially and keeps increasing to its long-run value of 3.3 percent. Initially, the increase in the growth rate of investment leads to a higher GDP growth rate. There are two countering forces that affect the investment growth rate. Discounted capital stock goes down under the social planner, which leads to a reduction in investment. However, because of an increase in the growth rate of the quality of investment goods, investment growth rate goes up. The second force dominates and we observe an increase in the growth rate of investment and hence GDP. Later on, as consumption and investment growth keeps increasing, the GDP growth rate converges to 3.3 percent. The transition from the market economy balanced growth path to the social planner equilibrium generates almost 15 percent welfare gain, measured in consumption equivalent terms.

Figure 2: GDP and Consumption Growth Rates, Social Planner



6 Innovation Subsidies

There is under-investment in innovation in both sectors as established in the previous section. Building on this result, I analyze the role of R&D subsidies in bringing the innovation rates to socially optimal levels and increasing welfare. I show that long-run welfare of the society can be increased substantially by providing R&D subsidies to incumbent and entering firms in each sector. Considering only time-invariant subsidies, welfare of the society can be increased by as much as 15 percent over the long-run.

In the previous section, various distortions of the economy are explained. My focus in this section is how the government can increase welfare by subsidizing innovation in both sectors, and at what rates the innovative activities in each sector should be subsidized. To answer these questions, I compare the welfare gains of various subsidy systems, which consist of the subsidy to capital investment, entry subsidy rates in each sector, and the incumbent firm R&D subsidy rates in each sector. The subsidy system is financed by lump-sum taxation of households. In all of the subsidy systems that I compare, the capital investment is subsidized with $1 - 1/\lambda_x$. This amount of subsidy offsets the distortion created by the monopoly pricing of investment good producers on the capital Euler equation. Also, in all of the subsidy systems, the entry subsidy rate is adjusted relative to the incumbent firm R&D subsidy rate to make innovative resource allocation within sectors across entering and incumbent firms to be optimal. Therefore, welfare comparisons of subsidy systems reflect welfare changes resulting from total innovation in that sector.

Starting from the balanced growth path of the benchmark economy (described in the calibration section), I alter the subsidy rates (unexpected to agents in the economy) for all the subsequent times and keep them constant. Then I calculate the transition to new balanced growth path under the new subsidy system. Afterwards, I calculate the welfare gain/loss of the subsidized economy relative to the benchmark economy. The algorithm I used to calculate the welfare impacts of the subsidy systems is described as follows.

1. Discount the variables that grow at the balanced growth path with the technology indices that leads to growth.
2. Solve for the steady states of the benchmark economy and subsidized economy.
3. Using the reverse shooting algorithm described by Judd (1998), solve the transition of the economy from the steady state of the benchmark economy to the steady state of the subsidized economy.
4. Starting from the steady state of the benchmark economy and by normalizing the technology indices at this steady state equal to one, simulate the economy forward and generate the consumption sequence (non-discounted). Attain two consumption sequences that will be used to compute welfare gain: 1) the consumption sequence of the benchmark economy, 2) the consumption sequence of the subsidized economy.

5. Calculate the sum of discounted utility of these two consumption sequences. Equation (25) is the closed form solution of the sum of the discounted utility of the benchmark economy which is at the balanced growth path, where C_0 is the consumption amount at the time of subsidy change and g_C is the consumption growth rate. The sum of discounted utility of the subsidized system is calculated using numerical integration over the utility values of consumption sequence,

$$W(C_0, g_C) = \frac{1}{\rho}\left(\ln C_0 + \frac{g_C}{\rho}\right). \tag{25}$$

6. Calculate the consumption equivalent welfare change described in Equation (26). The welfare gain/loss is equal to ξ: the rate of increase in consumption in the benchmark economy that will make the representative household indifferent with moving to the subsidized economy,

$$W(\xi C_0, g_C) = \int_0^\infty \exp(-\rho t)\ln(C_t^s)dt, \tag{26}$$

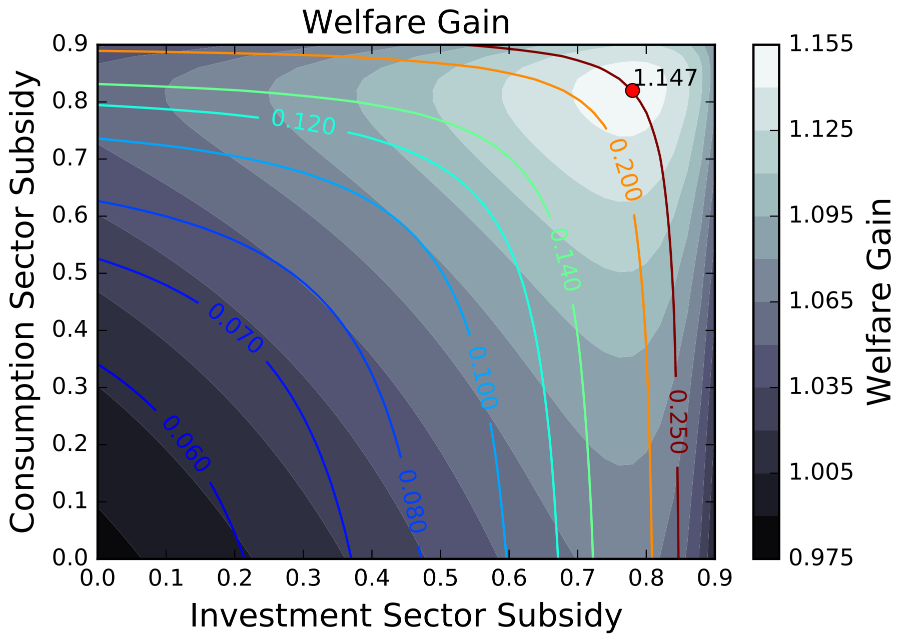
where C_t^s is consumption at time t in the subsidized economy.

I calculate the welfare gain of the subsidy systems in this set: $\{(1-1/\lambda_d, s_c, s_x, (s_c-\gamma)/(1-\gamma), (s_x-\gamma)/(1-\gamma)) : s_c = 0, .02, ..., .9, \quad s_x = 0, .02, ..., .9\}$. By considering subsidies from 0 percent to 90 percent, I cover all the relevant subsidy rates. The welfare gains of the subsidies in this set are depicted in Figure 3 as a contour map. The total amount of R&D expenditures these subsidies induce are shown in Appendix C

There are several results worth highlighting. First, holding subsidy of a sector constant as the subsidy of the other sector increases, so does the welfare gain until a certain point. Afterwards more subsidy results in a reduction in welfare gain. The same result holds when subsidies to both sectors increase simultaneously. Remembering the distortions identified in Section 5.2, the competitive equilibrium innovation rate can be above or below the social planner innovation rate. In this economy, it is below. Therefore, raising innovation rate to socially optimal levels leads to higher welfare. When the innovation rates surpass the optimal levels, welfare gains start decreasing. Maximum welfare gain is attained by subsidizing consumption sector R&D by 82 percent and investment sector by 78 percent. However, this result suggests that the level of under-investment in innovation is quite high for both sectors. Second, iso-welfare curves are tilted towards investment sector R&D subsidy. A given rate of subsidy generates higher welfare gain when it is applied to only investment sector than when it is applied only to the consumption sector.

There are two main reasons that correcting for the distortions requires approximately 80 percent R&D subsidy to each sector. First, as explained in the Section 5.3, there are large distortions in the economy. The amount of resources allocated to R&D under the social planner is more than the amount of innovative resources in the market economy. This high level of increase in resource allocation to innovation under the so-

Figure 3: Welfare Gain

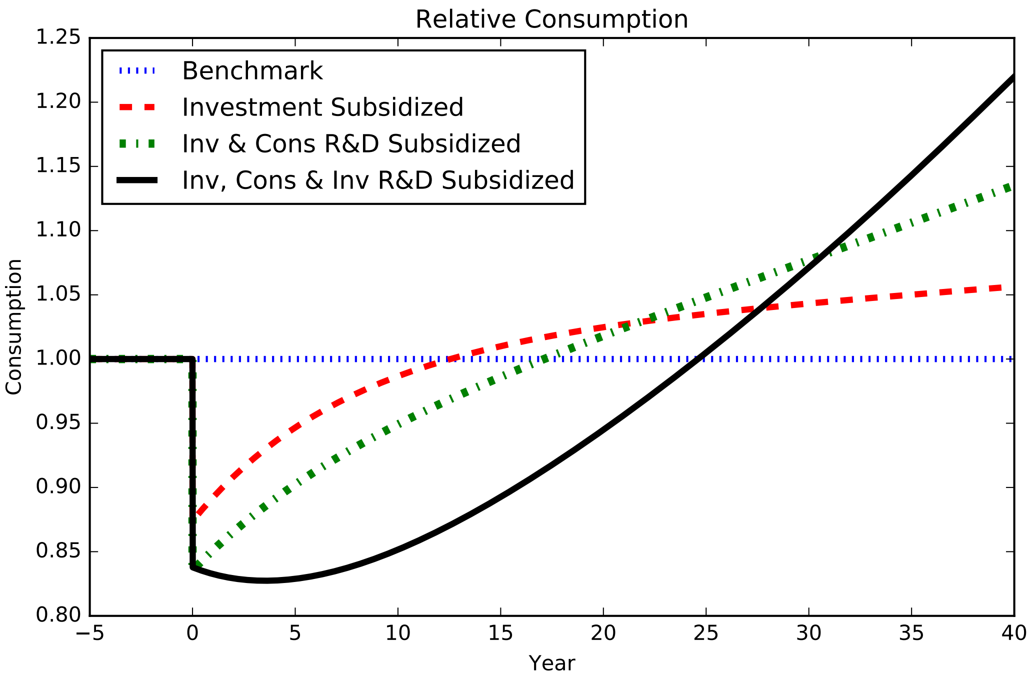


Notes: Contour map of welfare gains of R&D subsidies. Capital investment is subsidized as well. The curves on top of contour shades show the cost of subsidies at the balanced growth path as a share of GDP at the balanced growth path.

cial planner is common in the models based on Klette and Kortum. When the general model of Atkeson and Burstein (2015) is calibrated to resemble Klette and Kortum closely, the social planner increases resource allocation to innovation three times over (eleven times over with another calibration). Lentz and Mortensen (2015) also show that social planner increases innovative resources threefold. Similarly, Segerstrom (2007) find that innovation should be heavily subsidized. Second, subsidizing R&D also promotes a higher entry rate by increasing the value of firms. The higher entry rate corresponds to a higher probability for an incumbent firm to shrink by one good. In other words, inter-temporal spillover effect increases which decreases incumbent firms incentive to innovate. To compensate for the higher inter-temporal spillover, firm R&D needs to be subsidized even more.

How does this economy achieve the maximum welfare gain? Analyzing the trajectory of consumption helps us to answer this question. Figure 4 shows the trajectories of consumption in the benchmark economy, when only capital investment is subsidized, an 82 percent consumption sector R&D subsidy is added on top of the capital investment subsidy, and a 78 percent investment sector R&D subsidy is added on top of all the other subsidies. For better comparison of consumption paths after the subsidy to the benchmark economy, I discounted each consumption path in the figure with the benchmark economy consumption. Allocating more research labor to innovation results in reduction in consumption goods production in earlier periods but a higher long-run consumption growth rate. Consumption in the consumption sector subsidized economy rebounds more quickly. However, consumption in the investment sector subsidized economy surpasses the consumption subsidized economy in later years. The reason why consumption grows more slowly in earlier periods with the investment sector subsidy lies in the response of capital to subsidies. Subsidizing investment sector R&D leads to higher innovation rates in this sector. This leads to a lower growth rate of the price of investment goods (higher in absolute terms) and higher user cost of capital. Therefore, capital accumulates slowly. Hence, in earlier years consumption grows at a lower rate when investment sector is subsidized. Later on, after the economy reaches the balanced growth path, the higher innovative step of investment sector generates a higher consumption growth rate. Therefore, consumption in this economy catches and surpasses the benchmark and consumption sector subsidized economy.

Figure 4: Sequence of Consumption with Different Subsidies

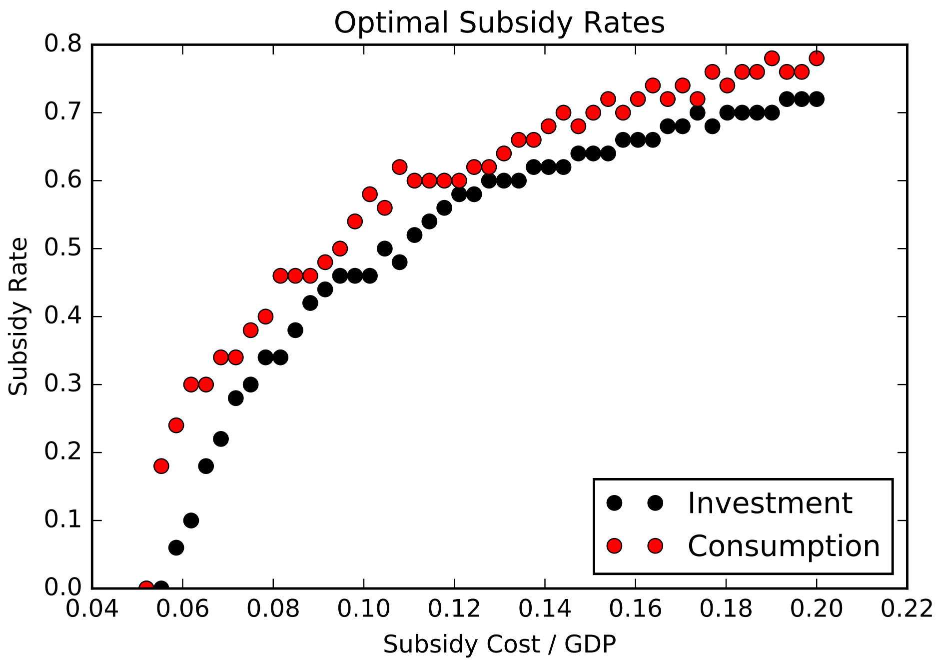


Notes: Consumption paths are relative to benchmark economy consumption level. Investment Subsidized: Benchmark with investment subsidy ($s_{in} = 1 - 1/\lambda_x$) added, Inv and Cons R&D Subsidized: Investment Subsidized with Consumption R&D subsidy of 78%, Inv, Cons & Inv R&D subsidized: The subsidy system that maximizes the welfare gain

6.1 Welfare Gain with Limited Transfer Budget

The amount of tax collection required to subsidize the economy to reach peak welfare gain is more than 25 percent of GDP. This amount is unreasonable because of two issues that are not modeled in this paper: distortionary effects of taxation and the political economy of taxation. Therefore, a related question is how the fiscal authority should allocate subsidies across sectors if its transfer budget is limited by some factors outside of the model. In this regard, I added iso-cost curves (total subsidy as a share of GDP on the balanced growth path) to Figure 3. Comparison of intersection of iso-cost curves with the graph axises reveal that it is more costly to subsidize the investment sector. This is mainly a result of higher R&D cost function parameters in the investment sector. Therefore, by allocating a higher subsidy rate to the consumption sector, a fiscal authority can increase the innovation rate in the consumption sector without decreasing innovation in investment sector as much. Comparison of this result with the above one introduces an interesting trade-off. On the one hand, a given rate of investment sector subsidy leads to higher welfare gain than an equal rate of consumption sector subsidy. On the other hand, a given rate of investment sector subsidy costs more than an equal rate of consumption sector subsidy. This trade–off determines the optimal allocation of limited transfer budget. In this economy, the cost advantage of the consumption sector dominates.

Figure 5: Optimal R&D Subsidy

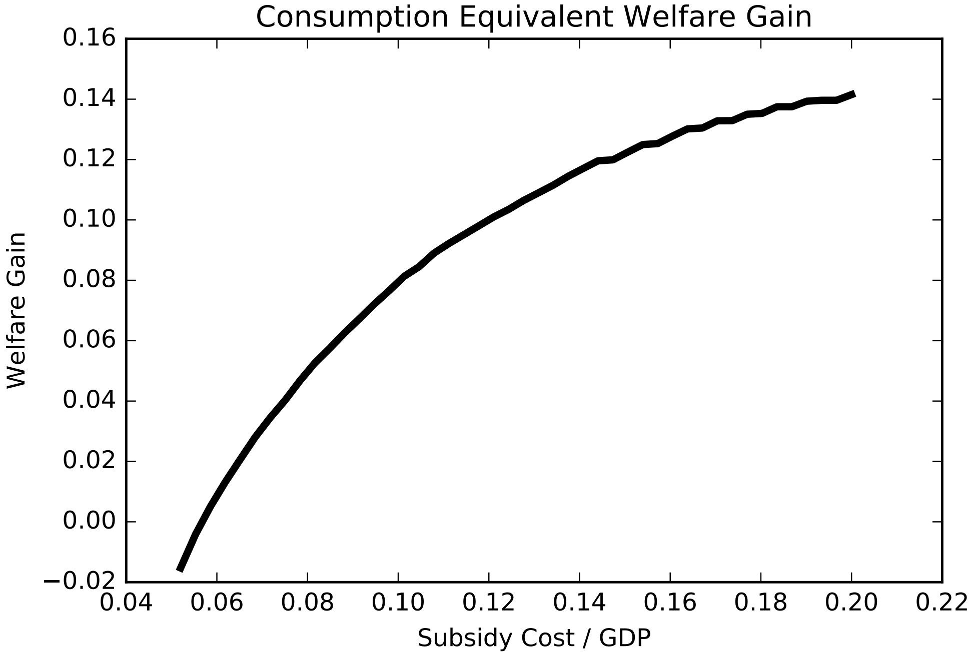


Notes: Optimal R&D subsidies to sectors under limited transfer budget.

Figure 5 shows optimal government R&D subsidy rate to sectors for different R&D subsidy cost over GDP ratios at the balanced growth path. It is always optimal to subsidize consumption sector at a higher rate than the investment sector. For example, if the tax authority has a transfer budget of 6.5 percent of GDP (including the capital investment subsidy), it is optimal to subsidize the consumption sector by 30 percent and the investment sector by 18 percent. Figure **??** shows welfare gain of optimal R&D subsidies under limited transfer budget. As the total amount of government budget allocated to R&D subsidies increases, the welfare gain in consumption equivalent terms increase at a decreasing rate.

There is one caveat, however. In my model, there is perfect separation of consumption and investment goods. In reality, a product can be both consumed by households and invested as capital, computers for example. Because of this, an innovation in

Figure 6: Welfare Gain



Notes: Welfare gain that can be achieved under limited transfer budget.

investment goods has both a direct and indirect effect on the consumption growth rate. Therefore, the contribution of the investment sector to growth will be higher than before, and vice versa for the consumption sector. The perfect separation of use of goods, in this case, reduces the welfare gain of an investment sector subsidy. Hence, we can informally argue that the investment sector should be subsidized more than as suggested above.

7 Conclusion

I analyze heterogeneity of innovative activity across sectors in a quantitative environment where firm level innovation is the main driver of the long–run macroeconomic growth. I ask how a government that wants to increase welfare of the society through R&D subsidies should target different sectors on the economy. To answer this and related questions, I develop a quality ladder type of model based on the framework of Klette and Kortum (2004) that features two sectors: consumption goods producers and investment goods producers. These sectors differ mainly in their output's use, R&D cost functions, and quality ladder steps sizes. An industry is classified as a consumption goods industry if household consumption of the industry's output is bigger than investment and inventory allocation made from its output. It is classified as an investment goods industry if vice versa. I calibrate my model using its firm dynamics implications and US data on job creation and destruction. An interesting result of calibration is investment sector firms are more innovative, have a higher quality ladder step, but have a higher cost of innovation.

A sector's contribution to macroeconomic growth and welfare of the society depends on the sector's position in the supply chain of the economy, its innovation rate, and the quality increase (or cost reduction) of the goods after a successful innovation in the sector. Consumption sector innovation affects consumption growth directly, whereas investment sector innovation affects consumption growth indirectly through its effect on the capital stock of the economy. Also, the consumption sector generates more innovation than the investment sector. In this sense, the consumption sector contributes more to growth. However, the investment sector is more innovative. Once it innovates, it increases the quality of existing goods more than the consumption sector. The number of innovations, say on central processing units (CPUs), are lower than the number of innovations, say on restaurants. However, once a better CPU is developed, its quality increase is higher than the quality increase of better restaurant food. This last effect, higher innovativeness of investment sector dominates and this sector contributes more to long-run macroeconomic growth, more than 60 percent.

The Schumpeterian innovation process described in the model leads to various distortions in the economy and innovation rates in both sectors are lower than socially desirable levels. Therefore, government can increase the welfare of the society in the long run by subsidizing R&D. This welfare gain, in consumption equivalent terms, can reach up to 15 percent. A given rate of R&D subsidy to the investment sector generates more welfare gain than an equal amount of R&D subsidy to the consumption sector.

A more realistic policy question is how the government should allocate a limited transfer budget. Though the investment sector has higher innovativeness, innovation is costly in this sector. By decreasing the subsidy rate of the investment sector and allocating higher rates to the consumption sector can compensate for the lower innovativeness of consumption sector. In optimality, a subsidy system tilted toward the consumption sector generates more welfare gain than a uniform subsidy system with the same cost.

Many of the results rely on quality ladder steps, which are identified by four statistics: the consumption growth rate, the growth rate of the relative price of investment goods, and the innovation rates in each industry. Any mismeasurement of these statistics would lead to biased results. For example, if the growth rate of relative price of investment goods was affected by factors other than the quality increase, the results would not be accurate. Similarly, in the model, the only source of quality improvement is dedicated R&D activity. In this sense, innovation in my model is regarded in the broadest sense: any activity that leads to quality improvement is innovation. Therefore, this strong assumption also contaminates results.

Lastly, in the model, there is a perfect separation of consumption and investment goods. Not in reality. A computer, for example, is both a consumption and investment good. Therefore technological progress in investment goods would have both direct and indirect affects on consumption goods. This would make innovation in the investment sector even more important.

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A Industry Classification

- Consumption-type:
  - Retail trade, except for motor vehicles and motorcycles; repair of household goods
  - Hotels and restaurants
  - Finance, insurance, real estate and business services
  - Community, social, and personal services
- Investment-type:
  - Mining and quarrying
  - Manufacturing
  - Electricity, gas, and water supply
  - Construction
  - Wholesale trade and commission trade, except for motor vehicles and motorcycles
  - Transport, storage and communication

B Solution of the Model

The representative household maximization problem is described in Section 2.1. Consumption is a quality adjusted aggregation of differentiated consumption goods described in Equation (1). Since, I solve for a symmetric equilibrium and limit pricing is assumed, highest quality versions of each differentiated consumption product gets the same positive demand, and the lower quality versions have a demand of zero. This demand function is described in Equation (2). Then we simplify equation 1 into

$$C = \exp\left(\int_0^1 \ln\left(q(\omega)c(\omega)\right) d\omega\right), \tag{27}$$

where $q(\omega)$ is the highest quality level of product ω, and $c(\omega)$ is the consumption of product ω with highest quality. Also, using the fact that the production function of differentiated goods in a sector is identical, and symmetric demand, the labor hired and capital rented across differentiated goods is the same, the production for each differentiated unit becomes $c(\omega) = k_c^{\alpha} l_c^{1-\alpha}$, and k_c and l_c do not depend on the product. Therefore, the aggregate consumption function turns into

$$C = \exp\left(\int_0^1 \ln\left(q(\omega)k_c^{\alpha} l_c^{1-\alpha}\right) d\omega\right) \tag{28}$$

$$C = k_c^{\alpha} l_c^{1-\alpha} \exp\left(\int_0^1 \ln\left(q(\omega)\right) d\omega\right) \tag{29}$$

$$C = k_c^{\alpha} l_c^{1-\alpha} Q_c, \tag{30}$$

where $Q_c = \exp\left(\int_0^1 \ln\left(q(\omega)\right) d\omega\right)$ is the average quality in the consumption sector. Equation (30) will be used to determine the growth rate of consumption on the balanced growth path. Average price of the industry adjusted for the quality, on the other hand, is equal to

$$\begin{aligned} P_c &= \exp\left(\int_0^1 \ln \frac{p(\omega)}{q(\omega)} d\omega\right) && (31)\\ &= \exp\left(\int_0^1 \ln \frac{\frac{r^\alpha w^{1-\alpha}}{\alpha}}{q(\omega)} d\omega\right) && (32)\\ &= \frac{r^\alpha w^{1-\alpha}}{\alpha}\frac{1}{Q_C} && (33) \end{aligned}$$

Again, this is a result of identical innovative steps and identical production functions. I normalize the price of the consumption good to 1:

$$1 \equiv P_c = \lambda_c \frac{r^\alpha w^{1-\alpha}}{\alpha Q_c} \tag{34}$$

Similarly, investment is a quality adjusted aggregation of differentiated investment goods. Using the same arguments as above, the demand function of differentiated investment goods can be inserted into the investment aggregator and combined with the identical production functions of differentiated investment goods, aggregate investment is written as

$$X = k_x^\alpha l_x^{1-\alpha} Q_x, \tag{35}$$

where $Q_x = \exp\left(\int_0^1 \ln\left(q(\omega)\right) d\omega\right)$ is the average quality in the investment sector. Quality adjusted average price of investment good is also equal price of each differentiated good:

$$P_x = \lambda_x \frac{r^\alpha w^{1-\alpha}}{A Q_x}. \tag{36}$$

The two other first order conditions of the household problem, consumption Euler equation and no arbitrage condition, and laws of motion of capital and asset holdings close the consumer part of the model:

$$\begin{aligned} &\frac{\dot{C}}{C} + \frac{\dot{P}_c}{P_c} = R - \rho, && (37)\\ &r = (R + \delta - g_{P_x})(1 - s_{in})P_x, && (38)\\ &\dot{A} = RA + wL + rK - P_c C - (1 - s_{in})P_x X, && (39)\\ &\dot{K} = X - \delta K. && (40) \end{aligned}$$

Turning to the firm side, cost minimization problems of consumption and investment firms lead to

$$rk_c = wl_c \left(\frac{\alpha}{1-\alpha}\right), \tag{41}$$

$$rk_x = wl_x \left(\frac{\alpha}{1-\alpha}\right). \tag{42}$$

And innovation decisions of firms in both sectors generates the following conditions

$$\chi_j \psi_j z_j^{\gamma/(1-\gamma)} = \frac{1}{1-\gamma} \chi_j b_j^{\gamma/(1-\gamma)}, \quad j = c, x, \tag{43}$$

$$(R + \tau_c - b_c) w \chi_c \psi_c z_c^{\gamma/(1-\gamma)} = \pi_c - w \chi_c b_c^{1/(1-\gamma)} + \frac{\partial V(1, Z)}{\partial Z} \dot{Z}, \tag{44}$$

$$(R + \tau_x - b_x) w \chi_x \psi_x z_x^{\gamma/(1-\gamma)} = \pi_x - w \chi_x b_x^{1/(1-\gamma)} + \frac{\partial V(1, I)}{\partial I} \dot{I}, \tag{45}$$

Adding the market clearing condition for labor closes the model:

$$L = l_c + l_x + \sum_{j=c,x} \chi_j \psi_j z_j^{1/(1-\gamma)} + \sum_{j=c,x} \chi_j b_j^{1/(1-\gamma)}. \tag{46}$$

B.1 Balanced Growth Path

To find the growth rates of the variables on the balanced growth path, suppose there exists such a path and then verify it. Let $g_a \equiv \frac{\dot{a}}{a}$ denote the growth rate of any variable a on the balanced growth path. Let Y denote the GDP of the economy, $Y = C + P_x X$. Then the growth rate of consumption is equal to growth rate of investment expenditures, $g_C = g_I = g_{P_x} + g_X$. Using the income approach to GDP, $Y = rK + wL + RA - \dot{A}$, the growth rate of consumption is equal to growth rate of the wage rate, $g_C = g_w = g_r + g_K$. Since the price of consumption is normalized to 1, then Equation (34) implies that $g_{Q_c} = \alpha g_r + (1-\alpha) g_w$. Using the investment price formula, Equation (36), $g_{P_x} + g_{Q_x} = \alpha g_r + (1-\alpha) g_w$. But, in order for the no arbitrage condition to hold, Equation (38), the growth rate of rental rate of capital should be equal to the growth rate of the relative price of investment goods, $g_r = g_{P_x}$. Then, putting the growth rate of consumption and investment price equations together,

$$g_{Q_c} = \alpha g_r + (1-\alpha) g_w,$$
$$g_{Q_x} = (\alpha - 1) g_r + (1-\alpha) g_w,$$

the growth rate of the wage and rental rates of capital can be solved as $g_w = g_{Q_c} + \frac{\alpha}{1-\alpha} g_{Q_x}$, and $g_r = g_{Q_c} - \frac{1-\alpha}{1-\alpha} g_{Q_x}$.

Now, by using the other equilibrium conditions, I can verify that the above growth rates are indeed the balanced growth path rates. First, the growth rate of consumption should be equal to $g_C = \alpha g_K + g_{Q_c}$ by Equation (30):

$$g_K + g_{Q_c} = g_w - g_r + g_{Q_c} = \frac{1}{1-\alpha} g_{Q_x} + g_c,$$

where the right hand side of the equation is equal to the growth rate of wage which is equal to growth rate of the consumption. It is straightforward to verify that the other equilibrium conditions are satisfied as well.

B.1.1 Growth Rates of Average Quality Levels

In this economy innovations occur with a Poisson rate of τ. Hence, in a time interval of t, the probability of exactly m innovations occur is equal to $f(m,t) = \frac{(\tau t)^m \exp(-\tau t)}{m!}$. Assuming the law of large numbers holds, the probability of having exactly m innovations in a time interval is equal to measure of products that had m innovations in that interval [Grossman and Helpman (1991)]. Plugging this back into the average quality level equation,

$$\begin{aligned}
Q_t &= \exp\left(\int_0^1 \ln q(\omega) d\omega\right) \\
&= \exp\left(\sum_{m=0}^{\infty} f(m,t) \ln \lambda^m\right) \\
&= \exp\left(\ln \lambda \sum_{m=0}^{\infty} f(m,t) m\right) \\
&= \exp\left(\ln(\lambda)\tau t\right),
\end{aligned}$$

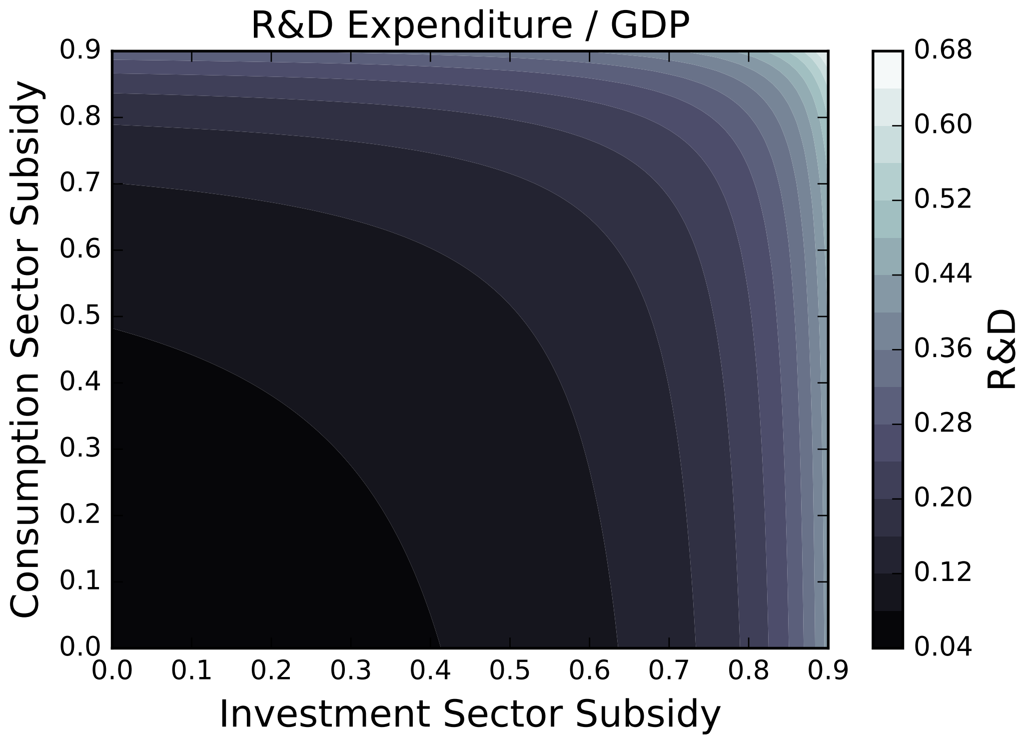
where the latter step is from the expectation of the Poisson distribution. Then the growth rate of average technology in each industry is equal to

$$\frac{\dot{Q}_j}{Q_j} = \tau_j \ln \lambda_j, \quad j = c, x. \tag{47}$$

C R&D Expenditure as a Share of GDP

This section shows the total R&D expenditures as a share of GDP at the balanced growth path.

Figure 7: Total R&D Expenditure as a Share of GDP



Notes: Contour map of R&D expenditure as a share of GDP at the balanced growth path.